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POSITIVE DEFINITE FUNCTIONS WHICH VANISH AT INFINITY

ALESSANDRO FIGÀ-TALAMANCA

Let G be a separable noncompact locally compact group. Let $A(G)$ and $B(G)$, respectively, be the Fourier algebra and the Fourier-Stieltjes algebra of G as defined by P. Eymard. We prove that if G is unimodular and satisfies an additional hypothesis, which implies noncompactness, there exists an element of $B(G)$, indeed a positive definite function, which vanishes at infinity but is not in $A(G)$. This function actually belongs to $B^\rho(G)$, that is, it defines a unitary representation of G which is weakly contained in the regular representation.

We refer to [3] for the definitions and properties of $A(G)$, $B(G)$, $B_\rho(G)$ and of the related spaces $VN(G)$, $C^*(G)$ and $C^*_\rho(G)$.

We recall only that $C^*_\rho(G)$ and $VN(G)$ are, respectively, the C^*-algebra and the von Neumann algebra, generated by $L^1(G)$ acting by left convolution on $L^2(G)$. While $C^*(G)$ is the C^*-algebra of the group obtained by completing the algebra $L^1(G)$ with respect to the norm $\|f\|C^*(G) = \sup_{\pi\in\Sigma}\|\pi(f)\|$, where Σ is the space of all $*$-representations of $L^1(G)$ as an algebra of operators on a Hilbert space. We also recall that $B(G)$ is, in a natural fashion, the dual of $C^*(G)$, while $B_\rho(G)$ is the dual of $C^*_\rho(G)$ which is a quotient algebra of $C^*(G)$. Finally $VN(G)$ is the dual of $A(G)$;

When G is commutative and $\hat{G}$ is the character group of G, $A(G)$ and $B(G)$, respectively, coincide with the algebra of Fourier transforms of elements of $L^1(\hat{G})$ and the algebra of Fourier-Stieltjes transforms of bounded regular measures on $\hat{G}$.

Thus for G commutative our result reduces to the classical theorem which asserts that on any nondiscrete locally compact abelian group $\hat{G}$, one can construct a singular measure with Fourier-Stieltjes transform vanishing at infinity. This classical result was proved for the first time, for the case $\hat{G} = T$ and $G = Z$, by M. D. Menchoff [10].

When G is noncommutative the situation may be quite different: I. Khalil proved in [6] that if G is the affine group of the line, i.e., the group of transformations $x \to ax + b$ of $\boldsymbol{R}$ into $\boldsymbol{R}$, then $B(G) \cap C_0(G) = A(G)$.

Therefore some other hypothesis, in addition to noncompactness of G, is needed for our result to be true. In this paper we show that $A(G) \neq B(G) \cap C_0(G)$ provided that G is unimodular and in addition satisfies the following condition: