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NOETHERIAN FIXED RINGS

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One of the basic questions of noncommutative Galois theory is the relation between a ring R and the ring S fixed by a group of automorphisms of R. This paper explores what happens when the group is finite and the fixed ring S is assumed to be Noetherian. Easy examples show that R may not be Noetherian; however, in this paper it is shown that R is Noetherian with some rather natural assuptions. More precisely we prove the Theorem 2: Let S be a semiprime ring. Assume that G is a finite group of automorphisms of S and that S has no $|G|$-torsion. If S^G is left noetherian then S is left noetherian.

Theorem 2 answers a question raised by Fisher and Osterburg [4]. This result rests on calculations which can best be described as belonging to noncommutative Galois theory. The basic theorem here may be of independent interest.

THEOREM 1. *Let R be a semisimple artinian ring. If G is a finite group of automorphisms of R and $|G|$ is invertible in R then R is a finitely generated ring R^G-module.*

The proof of Theorem 1 follows the spirit of Karchenko's work on polynomial identity rings ([6]).

1. A proof of Theorem 1.

We will repeatedly need Levitzki's fixed ring theorem ([8]): Suppose R is a semisimple artinian ring. If G is a finite group of automorphisms of R with $|G|$ invertible in R then R^G is semisimple artinian.

LEMMA 1. *If Theorem 1 is true when G is a simple group then it is true for an arbitrary finite G.*

Proof. By induction on the length of a composition series for G. If G is not already simple choose $H \Delta G$ with $1 \neq H \neq G$. By Levitzki's theorem R^H is semisimple artinian. G/H acts on R^H and R^H has no $|G/H|$-torsion; by induction R^H is a finitely generated right R^G-module. Again, induction shows that R is a finitely generated right R^H-module. The lemma follows.

We eventually assume that G is simple. In that case either G consists entirely of outer automorphisms or entirely of inner automorphisms.