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DECOMPOSITIONS FOR NONCLOSED PLANAR m-CONVEX SETS

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Let S be an m-convex set in the plane having the property that (int cl S)$\sim S$ contains no isolated points. If T is an m-convex subset of S having convex closure, then T is a union of $\sigma(m)$ or fewer convex sets, where

$$\sigma(m) = (m-1)[1 + (2^{m-2} - 1)2^{m-3}] .$$

Hence for $m \geq 3$, S is expressible as a union of $(m-1)^3 2^{m-3}\sigma(m)$ or fewer convex sets.

In case S is m-convex and (int cl S)$\sim S$ contains isolated points, an example shows that no such decomposition theorem is possible.

1. **Introduction.** For S a subset of Euclidean space, S is said to be m-convex, $m \geqq 2$, if and only if for every m distinct points of S, at least one of the line segments determined by these points lies in S. Several decomposition theorems have been proved for m-convex sets in the plane. A closed planar 3-convex set is expressible as a union of 3 or fewer convex sets (Valentine [4]), and an arbitrary planar 3-convex set is a union of 6 or fewer convex sets (Breen [1]). Concerning the general case, a recent study shows that for $m \geqq 3$, a closed planar m-convex set may be decomposed into $(m-1)^3 2^{m-3}$ or fewer convex sets (Kay and Breen [2]). This leads naturally to the problem considered here, that of determining whether such a bound exists for an arbitrary m-convex set $S \subseteqq R^2$: With the restriction that (int cl S) $\sim S$ contain no isolated points, a bound in terms of m is obtained; without this restriction, an example reveals that no bound is possible.

The following terminology will be used: For points x, y in S, we say *x sees y via S* if and only if the corresponding segment $[x, y]$ lies in S. Points $x_1, \cdots, x_n$ in S are *visually independent via S* if and only if for $1 \leqq i < j \leqq n$, x_i does not see x_j via S. Throughout the paper, conv S, bdry S, int S, and cl S will be used to denote the convex hull of S, the boundary of S, the interior of S and the closure of S, respectively.

2. **The decomposition theorem.** We shall be concerned with the proof of the following result, which yields the decomposition theorem as a corollary.