PACIFIC JOURNAL OF MATHEMATICS
Vol. 70, No. 1, 1977

EXPONENTIAL REPRESENTATION OF SOLUTIONS TO AN ABSTRACT SEMI-LINEAR DIFFERENTIAL EQUATION

G. F. WEBB

It is shown that the solutions to the abstract differential equation $u' = -(A+B)u$, $u(0) = x \in X$, where X is a Banach space, $-A$ is a linear analytic semigroup generator, and B is Lipschitz continuous from the domain of a fractional power of A to X, have the exponential representation $u(t) = \lim_{n\to\infty}(I + t/n\,(A+B))^{-n}x$.

1. Introduction. Let X be a Banach space with norm $\|\ \|$. We are concerned with the abstract semi-linear differential equation in X

$$du(t)/dt = -(A+B)u(t), \quad t > 0, \quad u(0) = x \in X, \tag{1.1}$$

where $-A$ is the generator of an analytic semigroup of linear operators in X and B is Lipschitz continuous from the domain of a fractional power of A to X. The objective of this paper is to obtain the exponential representation of the solutions to (1.1) in the form

$$u(t) = \lim_{n\to\infty}(I + t/n(A+B))^{-n}x. \tag{1.2}$$

Exponential representations of the form (1.2) are very well known for the case that A and B satisfy accretive type conditions (see, e.g., [**1**] and [**8**]). In the accretive case the nonlinear resolvent $(I + t/n(A+B))^{-1}$ is Lipschitz continuous with

$$|(I + t/n(A+B))^{-1}|_{\mathrm{Lip}} \leqq (1 - t\gamma/n)^{-1}, \ t \geqq 0, \ n \quad \text{sufficiently large},$$

where γ is some real constant. In our case the main difficulty in establishing (1.2) is that the nonlinear resolvent satisfies a more general condition of the form

$$|(I + t/n(A+B))^{-n}|_{\mathrm{Lip}} \leqq M(1 - t\gamma/n)^{-n}, \ t \geqq 0, \ n \quad \text{sufficiently large},$$

where M and γ are real constants and $M > 1$.

We make the following assumption on A: