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ENUMERATING NORMAL BUNDLES OF IMMERSIONS AND EMBEDDINGS OF PROJECTIVE SPACES

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All normal bundles of immersions $P^m \to R^{2m-\epsilon}$ for $m \geqq 7$, $\epsilon \leqq 2$ are classified. Those represented by embeddings are identified, and, for $\epsilon \leqq 1$, those which compress to an immersion $P^m \to R^{2m-\epsilon-1}$ are identified.

1. Introduction. The notation of [10] and [11] is used. Let all manifolds be differentiable, and all vector bundles real. Write P^m for real projective m-space, and let h be the canonical line bundle over P^m. If V is a manifold, let $[V \subseteq R^n]$ and $[V \subset R^n]$, respectively, be the set of regular homotopy classes of immersions $V \to R^n$ and the set of isotopy classes of embeddings $V \to R^n$. If ξ is a stable vector bundle over a complex X, let $A_k(X;\xi)$ be the set of equivalence classes of stabilized k-plane bundles over X representing ξ. (Equivalently, let $A_k(X;\xi)$ be the set of fiber-homotopy classes of liftings of the classification map $X \to BO$ to BO_k.) Thus, if $\dim V = m, [V \subseteq R^n] = A_{n-m}(V;\nu_V)$, where ν_V is the stable normal bundle of V.

Let $V_k(X;\xi)$ be the set of equivalence classes of k-plane bundles over X which represent ξ, i.e., the set of equivalence classes of $A_k(X;\xi)$ under ordinary bundle equivalence. There is a naturally defined action

$$\gamma\colon KO^{-1}(X) \times A_k(X;\xi) \to A_k(X;\xi)$$

such that the orbits of γ are precisely the elements $V_k(X;\xi)$; if $\dim X \leqq 2k-2$, $A_k(X;\xi)$ is an Abelian affine group and γ is an affine action, i.e., for any $\alpha \in KO^{-1}(X)$, $\gamma(\alpha, \ )\colon A_k(X;\xi) \to A_k(X;\xi)$ is an affine isomorphism. In that range, the so-called metastable range, let $A_k^0(X;\xi)$, an Abelian group, be the difference group of $A_k(X;\xi)$, provided the latter is nonempty. Any nonempty Abelian affine group is identified with its difference group by identifying some element with 0. This choice is arbitrary, and different choices may result in different expressions of the action γ and the corresponding equivalence relation on $A_k(X;\xi)$. The statements of theorems 1 and 2 are based on *some* choice.

Recall [11] that $[P^m \subseteq R^{2m-\epsilon}] \cong A_{m-\epsilon}(P^m, \nu)$, where ν is the stable normal bundle of P^m, and is an Abelian affine group (called the immersion group) if $m \geqq 7$ and $\epsilon \leqq 2$ (the only cases considered here). The orbits of the natural action of $KO^{-1}(P^m)$ on that immersion group