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LIPSCHITZ SPACES OF DISTRIBUTIONS ON THE SURFACE OF UNIT SPHERE IN EUCLIDEAN n-SPACE

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In this paper Lipschitz spaces of distributions are defined and various inclusion relations are shown. Certain properties such as completeness, separability, and the density of the testing space for appropriate Lipschitz spaces are proved. The Littlewood–Paley function is defined and used to prove inclusion relationships between Lipschitz and Lebesgue spaces.

This paper is the second in a series of papers by the author of which **[1]** will be used extensively in this paper. As a result, a knowledge of **[1]** would be useful to the reader. In **[1]** the discussion was limited to Lipschitz spaces of functions. Here we extend the definition of a Lipschitz space to Distributions.

Conventions and notation.

$\mathbf{R}^1$ will denote the real numbers.

$\mathbf{R}^n = \{x = (x_1, \cdots, x_n): x_i \in \mathbf{R}^1, i = 1, \cdots, n\}$.

$\Sigma_{n-1} = \{x \in \mathbf{R}^n : |x| = (x_1^2 + \cdots + x_n^2)^{1/2} = 1\}$. All functions are complex valued unless otherwise stated.

$C^\infty(\Sigma_{n-1})$ is the set of indefinitely differentiable functions on Σ_{n-1}.

All statements about continuity, bounded, finiteness, etc., are made modulo sets of measure zero unless otherwise specified. By this we mean that a function that can be modified on a set of measure zero to have the property will be said to have the property.

If $f(x, r)$, where $x \in \Sigma_{n-1}$ and $0 < r < 1$, is differentiable with respect to r, we define $Tf(x, r) = d/dr(rf)(x, r)$ and $T^k f(x, r) = T(T^{k-1}f)(x, r)$ where k is an integer greater than 1. We say $f(x) = 0(g(x))$, $x \to a$, if $f(x)/g(x)$ is bounded as $x \to a$.

$f(x) = o(g(x))$, $x \to a$, if $f(x)/g(x) \to 0$ as $x \to a$.

$f(x) \simeq g(x)$, $x \to a$, if $f(x)/g(x) \to 1$ as $x \to a$.

For α real, $\bar{\alpha}$ will denote the smallest nonnegative integer larger than α. If $f(x)$ is measurable on Σ_{n-1}, we define $\|f(x)\|_p = \left[\int_{\Sigma_{n-1}} |f(x)|^p\right]^{1/p}$, $1 \leqq p < \infty$, and $\|f(x)\|_\infty = \operatorname{ess\,sup}_{x \in \Sigma_{n-1}} |f(x)|$ where dx is nonnormalized Lebesgue measure on Σ_{n-1}. If $f(x, r)$ is measurable in x and r where $x \in \Sigma_{n-1}$ and $0 < r < 1$, we define