PACIFIC JOURNAL OF MATHEMATICS
Vol 70, No 1, 1977

FACTORIZATION OF RADONIFYING TRANSFORMATIONS

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It is shown that a linear transformation which carries a cylinder measure on a separable Hilbert space to a Radon measure on a separable Banach space can be factored into a positive-definite Hilbert-Schmidt transformation followed by a measurable linear transformation. Applications to measurable norms are given.

1. Introduction. It is well known that Hilbert–Schmidt transformations carry certain well behaved cylinder measures into Radon measures on Hilbert spaces (see [**8**, **9**]). The problem of characterizing transformations which carry cylinder measures on Hilbert spaces to Radon measures on Banach spaces seems to be more difficult (see [**8**]). In this paper it is shown that such a transformation can be factored into a positive-definite Hilbert–Schmidt transformation followed by a measurable linear transformation. In the last section this type of factorization is applied to abstract Wiener spaces.

2. Radon measures on embeddable spaces. Throughout this paper all topological vector spaces (TVS) will be assumed to be real and locally convex. A Radon measure is a regular Borel measure, and we shall assume that all Radon measures in this paper are positive with total measure 1, i.e. they are probability measures. A topological space X is a *Lusin space* if there is a complete separable metric space Y and a continuous bijective mapping $Y \to X$. Any Borel measure on a Lusin space is a Radon measure ([**9**], p. 122). The Borel subsets for comparable Lusin topologies are identical ([**9**], p. 101).

DEFINITION 2.1. A TVS E is *embeddable* if E is a Lusin space and if there is a continuous linear injection $T: E \to H$ where H is a separable Hilbert space.

Any such mapping T will be called an embedding of E. We can assume that $T(E)$ is dense in H. Kuelbs [**4**] has shown that any separable Banach space is embeddable. In fact we have

LEMMA 2.2. *A TVS is embeddable if and only if it is a Lusin space and there exists a countable bounded set* $\{y_i\} \subset E'$, *the dual of* E, *which separates points on* E.