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REMARKS ON SINGULAR ELLIPTIC THEORY FOR COMPLETE RIEMANNIAN MANIFOLDS

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This paper is about a C^*-algebra $\mathfrak{A}$ of 0-order pseudo-differential operators on $L^2(\Omega)$, where Ω is a complete Riemannian manifold which need *not* be compact. This algebra is designed to handle singular elliptic theory for certain Nth-order differential operators. In particular, this paper studies the maximal ideal space M of the (commutative) algebra $\mathfrak{A}/\mathcal{K}$, where $\mathcal{K}$ denotes the compact ideal. The space M contains the bundle of cospheres as a subspace, and in general will contain additional points at infinity of the manifold. The significance of this for elliptic theory lies in the fact that an operator $A \in \mathfrak{A}$ is Fredholm if and only if the associated continuous function $\sigma_A \in C(M)$ is never zero.

1. Introduction. Let Ω be an n-dimensional paracompact C^∞-manifold with *complete* Riemannian metric $ds^2 = g_{ij}dx^i dx^j$ and surface measure $d\mu = \sqrt{g}\, dx$ where $g = \det(g_{ij})$. As in **[5]** we define $\Lambda = (1-\Delta)^{-1/2}$ as a positive-definite operator in $\mathcal{L}(\mathfrak{h})$, the bounded operators over the Hilbert space $\mathfrak{h} = L^2(\Omega, d\mu)$, and define the Sobolev spaces $\mathfrak{h}_N \subset \mathfrak{h}$ for $N = 0, 1, \cdots$ by requiring $\Lambda^N : \mathfrak{h} \to \mathfrak{h}_N$ to be an isometric isomorphism. It was shown in **[3]** that $C_0^\infty(\Omega)$ is then dense in each $\mathfrak{h}_N$.

In **[5]** we defined classes of bounded functions and vector fields, **A** and **D**, whose successive covariant derivatives with respect to a symmetric affine connection ∇ *vanish at infinity* in the special sense that for $f \in C(\Omega)$ we write $\lim_{x\to\infty} f = 0$ if for every $\epsilon > 0$ there exists a compact set $K \subset \Omega$ such that

$$|f(x)| < \epsilon \quad \text{for} \quad x \in \Omega \setminus K. \tag{1.1}$$

Let $\mathbf{L}^N$ denote the class of Nth-order differential operators generated by taking sums of products of elements in **D** and **A**. The connection ∇ need not be the Riemannian connection ∇g, but must satisfy *Condition* (r_0) of **[5]** that it does not differ drastically from ∇g at infinity. We also require *Condition* $(\mathbf{L}^2)$ that 1-$\Delta \in \mathbf{L}^2$, a condition which was seen in **[5]** to imply the curvature tensor R tends to zero as $x \to \infty$ in the sense of (1.1). Under these two conditions it was shown that the operators $L\Lambda^N$ and $\Lambda^N L$ for $L \in \mathbf{L}^N$ are bounded over $\mathfrak{h}$ and thus generate an algebra $\mathfrak{A}^0 \subset \mathcal{L}(\mathfrak{h})$. Moreover it was found that after adding the compact ideal $\mathcal{K}$ to