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SEMI-SIMPLE CLASSES IN CHEVALLEY TYPE GROUPS

N. BURGOYNE AND C. WILLIAMSON

A practical method is given for finding the classes and centralizers for arbitrary p'-elements in the automorphism group of a Chevalley type group over a field of characteristic p.

1. Introduction. In the study of finite simple groups it is important to know their conjugacy classes and the structure of the corresponding centralizer subgroups. For the alternating groups the results are well known; for the sporadic groups the calculations are special to each group. In this article the authors will study the semi-simple classes in Chevalley type groups. Different methods are required for their unipotent classes.

Our approach is to work, as far as possible, in the algebraic group G corresponding to the given finite Chevalley type group. If t is a semi-simple element in G then, in general, $C_G(t)$ is not connected and its component of the identity is not semi-simple but only reductive. Since certain applications **[3]** require the structure of centralizers of pairs of commuting semi-simple elements we are led to study the rather general situation described in §2, 3, and 4. The underlying theory for these sections is quite simple and is based on essentially two results; (i) the algorithm which leads to the fundamental domain $\mathscr{F}_d$ of 2.4, for this see **[2]**, **[14]** or Appendix 2, and (ii) a general result about algebraic groups, see **[14**, §7**]**, which allows one to reduce questions about semi-simple elements in G to linear algebra problems in certain lattices. In a given case, once the situation in G is clear, the step down to the finite group is easily done by application of Lang's theorem, see **[11]**, **[13]** and §5 below.

In two unpublished notes **[4]**, **[5]** this approach was used to calculate (i) the classes of involutions at odd characteristic and (ii) the 3-elements at even characteristic in $\mathrm{Aut}(L)$ for all finite Chevalley type groups L. We also described the layer of $C_L(t)$. Rather than reproduce these results, we include, in Appendix 1, the structure of certain centralizer subgroups that are of interest for current work on simple groups of component type, see for example **[7]**, **[9]**.

We are indebted to the fundamental paper of Steinberg **[14]** for the basic theory. Earlier work on these question occurs in Abe **[1]**, Ree **[12]** and Iwahori **[10]**. In fact the starting point for our work was the attempt to put the ideas of **[10]** in a form which would give rapid and explicit answers to the sort of questions which arise from finite group theory.

Our notation for finite groups follows **[8]** and for algebraic groups **[6]**