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THE CONSTRUCTIVE RADON–NIKODYM THEOREM

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This paper discusses absolute continuity of integrals, and proves a version of the Radon–Nikodym Theorem and its converse, within the framework of the constructive measure theory of Bishop and Cheng.

1. Introduction. There can be few mathematicians who remain unaware of the fact that much of their subject, as commonly presented, has little or no computational significance. It will be convenient to refer to such mathematics as 'classical', in contrast to the alternative 'constructive' mathematics, in which

'*every mathematical statement ultimately expresses the fact that if we perform certain computations within the set of positive integers we shall get certain results.*' [**1**, p. 2].

In other words, the constructive philosophy (that adopted throughout this paper) insists that mathematics should be characterized by *numerical content* and *computational method.*

A simple consequence of our philosophy is the recognition of the familiar 'least upper bound principle' as an essentially nonconstructive proposition: an algorithm for computing the suprema even of sequences in $\{0, 1\}$ would provide at a stroke a method for deciding virtually all the outstanding unsolved problems of number theory [**1**, pp 6–7]. The effects of this situation appear throughout the development of constructive analysis. Thus, for example, we have no guarantee that the norm of a given bounded linear functional on a normed linear space will be computable (if it is, we call the functional *normable*); this means that we have to adopt the following as our constructive version of

THE RIESZ REPRESENTATION THEOREM. *A bounded linear functional f on a Hilbert space H is normable if and only if there exists a (unique) element ξ of H such that $f(x) = (x \mid \xi)$ for each x in H.*

The current revival of interest in the practice of constructive mathematics is due largely to Bishop, in whose fundamental book **[1]** there is developed a substantial portion of constructive functional analysis and measure theory. More recently, Bishop and Cheng have produced a much more elegant, and surely definitive, treatment of