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A COMMUTATIVITY STUDY FOR PERIODIC RINGS

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Putcha and Yaqub have proved that a ring R satisfying a polynomial identity of the form $xy = \omega(x, y)$, where $\omega(X, Y)$ is a word different from XY, must have nil commutator ideal. Our first major theorem extends this result to the case where $\omega(X, Y)$ varies with x and y, with the restriction that all $\omega(X, Y)$ have length at least three and are not of the form X^nY or XY^n. Further restrictions on the $\omega(X, Y)$ are then shown to yield commutativity of R; among these is a semigroup condition of Tamura, Putcha, and Weissglass—sepecifically, that each $\omega(X, Y)$ begins with Y and has degree at least 2 in X. The final theorem establishes commutativity of rings R satisfying $xy = yxs$, where $s = s(x, y)$ is an element in the center of the subring generated by x and y. All rings considered are either periodic by hypothesis or turn out to be periodic in the course of the investigation.

1. Definitions and preliminary results. Let $\omega = \omega(X, Y)$ be a word or monomial in the noncommuting indeterminates X and Y; that is, ω is a polynomial of form

$$Y^{j_1}X^{k_1}Y^{j_2}X^{k_2}\cdots Y^{j_s}X^{k_s}, \tag{1}$$

where $j_i, k_i \geqq 0$ for $i = 1, \cdots, s$ and $\Sigma_{i=1}^{s}(j_i + k_i) > 0$. By the X-length (resp. Y-length) of ω, which we denote by $|\omega|_X$ (resp. $|\omega|_Y$), we shall mean the non-negative integer Σk_i (resp. Σj_i); the sum $|\omega|_X + |\omega|_Y$ will be called the length of ω and denoted by $|\omega|$. It will be convenient to divide the set of all words into nine types as follows:

(i) words with $|\omega|_X \geqq 2$ and $|\omega|_Y \geqq 2$;
(ii) words of form $YX^n, n \geqq 1$;
(iii) words of form $Y^nX, n \geqq 1$;
(iv) words with $|\omega|_Y = 0$;
(v) words with $|\omega|_X = 0$;
(vi) words of form $X^nYX^m, n, m \geqq 1$;
(vii) words of form $Y^nXY^m, n, m \geqq 1$;
(viii) words of form $X^nY, n \geqq 1$;
(ix) words of form $XY^n, n \geqq 1$.

A word of form (1) having $j_1 \geqq 1$ and $|\omega|_X \geqq 2$ will be called a Tamura-Putcha-Weissglass (T-P-W) word; a word which is either YX or a T-P-W word will be called a G-T-P-W word. A multiplicative semigroup S will be called a T-P-W (resp. G-T-P-W) semigroup if for