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FUNCTIONS ACTING IN WEIGHTED ORLICZ ALGEBRAS

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Any complex valued function F with $F(0)=0$, which is Lipschitz continuous at 0 operates on all weighted Orlicz sequence algebras. If the weight increases sufficiently rapidly the class of functions which operate is strictly larger than the above class.

In this note we investigate the functional calculus of weighted Orlicz sequence algebras. We show that such an algebra has non Lipschitz continuous functions operating on it, if the weight increases sufficiently rapidly. On the other hand one knows: Let $\mathscr{A}$ be a commutative semisimple complex completely regular Banach algebra with identity and hermitian involution. Assume a function $F\colon(-1,1)\to\mathbf{R}$ with $F(0)=0$ and $\overline{\lim}_{t\to 0}|F(t)/t|=\infty$ operates on $\mathscr{A}$. Then $\mathscr{A}$ is the algebra of all continuous functions on its spectrum [**1**, Corollary 8.5]. This note was motivated by a paper of F. Gulick [**2**], who investigated the functional calculus of commutative $_*$-subalgebras of $\mathscr{C}_p(\mathscr{H})$, $1\leqq p<\infty$, the algebra of all compact operators x on some Hilbert space $\mathscr{H}$ with $|x|_p=(\mathrm{Tr}(x^*x)^{p/2})^{1/p}<\infty$.

Let $\mathscr{A}$ be a commutative $*$-subalgebra of $\mathscr{C}_p(\mathscr{H})$, $1\leqq p<\infty$, for some Hilbert space $\mathscr{H}$. By the spectral theorem the elements of $\mathscr{A}$ can be diagonalized simultaneously, i.e. there exists a sequence of finite dimensional projections $(P_i)_{i\in I}$, such that each $x\in\mathscr{A}$ can be written as $x=\Sigma\,\lambda_i(x)P_i$ and $|x|=(\mathrm{Tr}(x^*x)^{p/2})^{1/p}=(\Sigma\,|\lambda_i(x)|^p\dim P_i)^{1/p}$. Clearly the spectrum of $\mathscr{A}$ can be identified with I. The Gelfand representation of $\mathscr{A}$ leads then to the following class of Banach algebras. Let I be a set and e a real valued function on I with $e(i)\geqq 1$, the weight function. Let $\mathscr{A}=l_{p,e}(I)$, $1\leqq p<\infty$ be the system of all complex (real) valued functions x on I with $|x|_p=(\Sigma\,|x(i)|^pe(i))^{1/p}<\infty$. Such an algebra one may call a weighted l^p-algebra. Hence in [**2**] Gulick actually studied the functional calculus of weighted l^p-algebras. It is natural to investigate the problem of the functional calculus in the context of the larger class of Orlicz sequence algebras, since it is essentially determined by the weight (Lemma 4) and depends to a lesser extent on the Orlicz function. Our results are considerable extensions of those in [**2**, §5], even in the case of weighted l^p-algebras.

Let M be a continuous nondecreasing convex function on $[0,\infty)$ with