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NON-CONTINUOUS DEPENDENCE OF SURFACES OF LEAST AREA ON THE BOUNDARY CURVE

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In this paper, we consider the question of continuous dependence associated with the following version of Plateau's problem: Given a (sufficiently smooth) Jordan curve Γ, find a surface of least area bounded by Γ. In other words, we ask whether a surface S_Γ of least area among surfaces bounded by Γ can be found, continuously in Γ. The answer is no; in fact, sometimes one does not even have local continuous dependence. That is, for certain curves Γ_0, one cannot find S_Γ continuously in Γ, even on any neighborhood of Γ_0.

We became interested in this question through our interest in constructive aspects of Plateau's problem. A general principle is that if a problem can be constructively solved, then the solution depends (at least locally) continuously on the parameters provided the parameters come from a complete metric space). This may be seen intuitively as follows: if a computer is to compute the solution to accuracy ϵ, it must be able to do so on the basis of an approximate value of the parameters. This is closely related to Hadamard's formulation of the concept of a "well-posed problem". In **[1]** we have made a metamathematical study of this "Principle of Local Continuity". It follows from the results there, together with the main theorem of this paper, that no proof of Plateau's problem as stated above (say for $C^{(n)}$ boundaries) can be given in known constructive formal systems; see **[2]** for details.

There is a weaker form of Plateau's problem, in which it is required to find, not a surface of least area, but a surface which is a critical point of the area functional; that is to say, a minimal surface. It was proved by Hildebrant and again by Tomi **[8]** that for analytic boundary curves, this problem can be locally continuously solved. This lends evidence to support the conjecture that this version of Plateau's problem can be constructively solved, at least for sufficiently smooth boundaries. It is almost certainly true that one does not have global continuous dependence of a minimal surface on the boundary curve, even though one does have local continuous dependence, but this has yet to be proved.[1]

For the sake of precision, we state exactly what is meant by "local

[1] Added in proof: This follows from the results of A. J. Tromba and the author, the cusp catastrophe of Thom in the bifurcation of minimal surfaces, to appear.