PACIFIC JOURNAL OF MATHEMATICS
Vol. 70, No 1, 1977

NOETHER'S THEOREM FOR PLANE DOMAINS WITH HYPERELLIPTIC DOUBLE

WILLIAM H. BARKER II

This paper is motivated by the observation that Noether's theorem for quadratic differentials fails for hyperelliptic Riemann surfaces. In this paper we provide an appropriate substitute for Noether's theorem which is valid for plane domains with hyperelliptic double. Our result is somewhat more explicit than Noether's, and, in contrast with the case of nonhyperelliptic surfaces, it provides a basis for the (even) quadratic differentials which holds globally for all domains with hyperelliptic double. An important fact which plays a significant role in these considerations is that no two normal differentials of the first kind can have a common zero on a domain with hyperelliptic double.

Let W be a closed Riemann surface of genus $g \geqq 1$. We wish to consider the class of analytic quadratic differentials on W. In terms of a local parameter $z = z(p)$, $p \in W$, recall that an analytic quadratic differential has the representation $f(z)dz^2$, where $f(z)$ is a regular analytic function of the variable z. The analytic quadratic differentials form a complex linear space of dimension $3g-3$ for $g \geqq 1$ and of dimension 1 for $g = 1$. Additionally, the product of two Abelian differentials of the first kind is an analytic quadratic differential. Noether's theorem is fundamental in that it provides for a basis for the analytic quadratic differentials on a nonhyperelliptic Riemann surface in terms of products of Abelian differentials of the first kind. See, for example, Hensel and Landsberg **[4]**, p. 502.

In contrast, it is a direct computation that on a hyperelliptic Riemann surface of genus $g \geqq 1$, the products $\theta_i\theta_j$ of Abelian differentials of the first kind span a complex linear space of dimension $2g-1$. Thus, in particular, Noether's theorem fails in the case of hyperelliptic surfaces of genus $g \geqq 3$. In this paper we obtain an appropriate substitute for Noether's thoerem in the case of hyperelliptic surfaces obtained as doubles of plane domains.

DEFINITION 1. For each $n \geqq 1$ let $\mathscr{A}_n$ be the class of all domains whose boundary is formed by n disjoint piecewise analytic curves. We shall denote by H_n the class of all domains in $\mathscr{A}_n$ which possess a hyperelliptic double, and by Σ_n the class of all domains which are the exterior of a system of n slits taken from the real axis.