PACIFIC JOURNAL OF MATHEMATICS
Vol. 70, No. 2, 1977

ON THE PRIME DIVISORS OF ZERO IN FORM RINGS

L. J. RATLIFF, JR.

A number of new results concerning the prime divisors of zero in the form ring $\mathscr{F}(A, I)$ of a Noetherian ring A with respect to an ideal I in A are found by using certain auxiliary rings of the Rees ring $\mathscr{R}(A, I)$. Then it is shown that in every semi-local ring R there exist open ideals Q such that the prime divisors of $u\mathscr{R}(R, Q)$ have a number of properties known to hold when Q is a normal ideal and R is analytically unramified.

1. Introduction. Form rings (= associated graded rings) have been studied in a number of important papers (such as [**3**], where the concept was introduced, and [**7, 15, 16**]) and many textbooks on commutative algebra (such as [**1, 5, 21**]). Such rings are an important tool in many investigations in commutative algebra, and therein they appear in an auxiliary role. Our concern in this paper is with a certain aspect of the form rings themselves, namely, their prime divisors of zero. It is important to know about these prime ideals for a number of reasons, such as: they can be used to help determine when a local ring is quasi-unmixed [**10**, Theorem 3.8(2)]; unmixed [**5**, (25.1)]; analytically irreducible [**5**, (25.15)]; or, analytically normal [**5**, (25.15)]. (Also, if I and K are ideals in a Noetherian ring A and $\bar{K}$ is the I-form ideal of K in $\mathscr{F} = \mathscr{F}(A, I)$ (the form ring of A with respect to I), then $\mathscr{F}/\bar{K} \cong \mathscr{F}(A/K, (I + K)/K)$, so knowledge about prime divisors of zero in form rings immediately gives knowledge about the prime divisors of $\bar{K}$.)

It turns out that, due to an important result of D. Rees [**15**, Theorem 2.1], the study of the prime divisors of zero in $\mathscr{F}(A, I)$ is equivalent to studying the prime divisors of the principal ideal $u\mathscr{R}$ in the Rees ring $\mathscr{R} = \mathscr{R}(A, I)$, since $\mathscr{F} \cong \mathscr{R}/u\mathscr{R}$ (and in this isomorphism, $\bar{K} \cong (KA[t, u] \cap \mathscr{R}, u)\mathscr{R}$). Now from this point of view, a number of auxiliary rings can be brought into play to help determine properties of the prime divisors of $u\mathscr{R}$ (and vice versa, these properties imply certain results for the auxiliary rings). (And because of this, the results in this paper are stated for the prime divisors of $u\mathscr{R}$, and the corresponding results for the prime divisors of zero in the form rings (or the prime divisors of $\bar{K}$) are not explicitly stated.)

The specific auxiliary rings which are of importance below are $\mathscr{R}(A, I^n)$ (for $n \geqq 1$), $\mathscr{R}(A^*, IA^*)$ (where A is semi-local and A^* is its