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ESSENTIAL SPECTRUM $\Gamma(\beta)$ OF A DUAL ACTION ON A VON NEUMANN ALGEBRA

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For a dual action β of a locally compact group G on a von Neumann algebra N we define the essential spectrum $\Gamma(\beta)$ as the intersection of all spectrum sp β^p of the restriction β^p of β to N_p when p runs over all nonzero projections in N^β. $\Gamma(\beta)$ is then an algebraic invariant for a covariant dual system $\{N, \beta\}$. $\Gamma(\beta)$ is a closed subgroup of G (Theorem 3.7). We introduce three kinds of concept for β such as integrable, regular and dominant (§§4, 5). The former concepts are weaker than the dominance. If β is regular, $\Gamma(\beta)$ coincides with the kernel of the action $\hat{\beta}$ on the center of the crossed dual product $N \otimes_\beta^d G$ (Theorem 6.1). If β is regular, $\Gamma(\beta)$ is normal and $\Gamma(\beta) = \Gamma(\tilde{\beta})$. If β is ergodic on the center $Z(N)$ and $\Gamma(\tilde{\beta}) = G$, then $N \otimes_\beta^d G$ is a factor and vice versa (Theorem 6.4). If β is regular, $\Gamma(\beta) = G$ is equivalent to $Z(N^\beta) \subset Z(N)$ (Proposition 6.3). If β is integrable on a factor N and if $\Gamma(\beta) = G$, then there is a lattice isomorphism between the closed subgroups of G and the von Neumann subalgebras of N containing N^β (Theorem 8.4). Moreover, by $N \otimes_\beta^d (H \backslash G)$ we mean the von Neumann algebra generated by $\beta(N)$ and $1 \otimes (L^\infty(G) \cap \lambda'(H)')$, where H is a closed subgroup of G and λ' is the right regular representation of G. $N \otimes_\beta^d (H \backslash G)$ coincides with the set of $x \in N \otimes_\beta^d G$ such that $\hat{\beta}_t(x) = x$ for all $t \in H$ (Theorem 7.2).

0. Introduction. In our previous paper [**17**, **16**, **21**] we have generalized Takesaki's duality to a general locally compact group in terms of a dual action and a crossed dual product as the following:

$$(M \otimes_\alpha G) \otimes_{\hat{\alpha}}^d G \sim M \otimes B(L^2(G))$$
$$(N \otimes_\beta^d G) \otimes_{\hat{\beta}} G \sim N \otimes B(L^2(G)).$$

In this paper we continue our study on dual actions and Takesaki's duality obtained in the above from the view point of covariant systems $\{M, \alpha\}$ and covariant dual systems $\{N, \beta\}$. Then we naturally raise some questions:

a. What is an invariant of equivalent covariant dual systems?

b. When does Takesaki's duality hold as a covariant (dual) system?

Using the spectrum of β given in [**17**, §5], we can define the essential spectrum $\Gamma(\beta)$ of β in §3 by the same manner as S set. Then $\Gamma(\beta)$ is a closed subgroup of G and an algebraic invariant of dual actions on a