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ON PRESERVATION OF E-COMPACTNESS

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In this paper we study preservation of E-compactness under taking finite unions (the finite additivity theorems of E-compactness) and under taking quotient images.

Throughout this paper spaces are assumed to be Hausdorff, and maps are continuous onto functions. Given a space E, we shall call a space X *E-completely regular* (*E-compact*) provided that X is homeomorphic to a subspace (respectively, closed subspace) of a product E^m for some cardinal m.

As far as additivity theorems are concerned, the first author has shown in **[1]** that *if a space X is normal and if it can be expressed as the union of a countable collection of closed R-compact spaces (R denotes the space of all real numbers), then X is R-compact.* The assumption that X is normal in the above theorem is essential. In fact, in **[2]**, **[4]** the first author has constructed an example of a completely regular, non-R-compact space X which can be expressed as the union of two closed R-compact subspaces. This example shows that finite additivity relative to closed subspaces fails for R-compactness. It can be shown that the same example satisfies the above statement with "R-compact" replaced by "N-compact". (N denotes the space of all nonnegative integers.) Using the same example it was shown that the image of an R-compact (N-compact) space under a perfect map need not be R-compact (respectively, N-compact). In **[4]**, some positive results in this direction have been obtained. The purpose of this paper is to generalize some of the results in **[4]** to a certain class of E-compact spaces which contains both the class of R-compact spaces and the class of N-compact spaces. Many theorems concerning the preservation of E-compactness can be stated in a more comprehensive form as rules concerning "E-defect" of spaces (for definition of E-defect, see next paragraph). In §2 we shall state the additivity theorems of E-compactness both in words and as rules concerning E-defects of spaces.

The reader is referred to **[3]** for basic results of E-completely regular spaces and E-compact spaces. For convenience we review the notations and terminology. Given two spaces X and E, $C(X, E)$ denotes the set of all continuous functions from X into E. A class $\mathscr{F} \subseteq C(X, E)$ is called an *E-non-extendable class* for X provided that there is no proper extension ϵX of X such that every $f \in \mathscr{F}$ admits a continuous extension $f^*\colon \epsilon X \to E$. The *$E$-defect* of a space X (in symbols, $\operatorname{def}_E X$) is the smallest (finite or infinite) cardinal p such that there exists an E-non-