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COMPLETENESS PROPERTIES FOR CONVERGENCE SPACES

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For uniform convergence spaces a completeness concept has been introduced by Cook and Fisher. For a uniformizable convergence space it is of interest whether among the uniform convergence structures inducing the given structure, one can find a complete one. Keller has shown that this is the case for every Hausdorff convergence space. Our aim is to introduce a stronger completeness concept. We have developed a theory in which completeness is a generalization of topological completeness for metrizable spaces.

We have based our theory on various completeness properties which have been introduced for topological spaces such as subcompactness, basecompactness and cocompactness **[13]**, **[22]**, **[1]**, **[3]**, **[4]**. A survey of those topological completeness properties is given by Aarts and Lutzer in **[6]**.

The crucial point in the introducing of completeness for convergence spaces was the definition of the concept of pointbases which for a convergence space plays the role of the fundamental systems of neighborhoods of a topology. Using this notion we shall introduce and study two completeness concepts.

The theory developed here corresponds to a part of the authors Thesis **[19]**.

For all notational conventions we refer to **[7]** and **[12]**. Definitions on convergence spaces that are used can be found in **[16]** and **[17]**. We recall those being used frequently.

Let X be a set and $x \in X$. The filter generated by $\{x\}$ is denoted by $\dot{x}$. If $\mathcal{G} \subset 2^X$ is a nonempty family with the finite intersection property, then $[\mathcal{G}]$ stands for the filter *generated* by $\mathcal{G}$.

A *convergence space* (X, q) is a set X together with a map q which to any point $x \in X$ assigns a family qx of filters on X. The filters in qx are said to converge to x. For every $x \in X$, qx contains $\dot{x}$ and for every filter $\mathcal{F}$ in qx the filter $\mathcal{F} \cap \dot{x}$ as well as every filter finer than $\mathcal{F}$ belongs to qx.

If in addition for every $x \in X$ the intersection of all the filters in qx belongs to qx then the convergence space is said to be *pretopological.*

With a convergence space (X, q) one associates a *pretopological* respectively *topological modification* denoted by $(X, \psi q)$ respectively $(X, \lambda q)$ **[16]**.