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SHEAR DISTALITY AND EQUICONTINUITY

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Let X be a compact Hausdorff space, let R^p be p-dimensional Euclidean space, and let (X, R^p) be a minimal transformation group. It may happen that $\overline{xH}$ will always contain flow lines in at least one direction in R^p for any discretr syndetic subgroup H no matter how sparce. We interpret this phenomena as some intrinsic shearing motion in the minimal transformation group. This is quantified in Section 1 and it turns out that equicontinuous minimal sets have as little shear as possible. Since distality is also a rigidity condition, it is natural to investigate the shear of a distal minimal set. We show by example in Section 2 that distal minimal sets can contain more shear than equicontinuous ones.

In Section 3 we show how the topology of $\overline{xH}$ is locally determined by local sections and subspaces of R^p. Using this result we prove in Section 4 that a distal minimal action of R^{n-1} with trivial isotropy on a compact n-dimensional manifold is equicontinuous.

This paper contains portions of the first author's dissertation [1] and generalizations of some results in an unpublished preprint [6] by the second author.

1. Shear. Let X be a compact Hausdorff space and let $(X, \boldsymbol{R}^p)$ be a minimal transformation group. Set $I_x = \{v \in \boldsymbol{R}^p : xv = x\}$ and note that it is a closed subgroup of $\boldsymbol{R}^p$ which is independent of x because of the minimality. This group will be denoted by I or $I(X, \boldsymbol{R}^p)$ and we will say that $(X, \boldsymbol{R}^p)$ has trivial isotropy when $I = \{0\}$. We will frequently need to assume that $(X, \boldsymbol{R}^p)$ is locally free; that is, given $x \in X$ there exists a neighborhood W of 0 in $\boldsymbol{R}^p$ such that $xv \neq x$ for all v in W. Clearly $(X, \boldsymbol{R}^p)$ is locally free if and only if I is discrete.

Let H be a closed syndetic (co-compact) subgroup of $\boldsymbol{R}^p$. It is well known that (X, H) is pointwise almost periodic and $H_x = \{v : xv \in \overline{xH}\}$ is also a closed syndetic subgroup of $\boldsymbol{R}^p$ such that $\overline{xH_x} = \overline{xH}$. [**4**, Theorem 4.04, Lemma 2.09 and Lemma 2.10.] Again by minimality H_x is independent of x. We will say that H is self-enveloping when $xv \in \overline{xH}$ implies $v \in H$ or $H_x = H$ for all x. When $\overline{xH} = X$ for every closed syndetic subgroup of $\boldsymbol{R}^p$, $(X, \boldsymbol{R}^p)$ is said to be totally minimal. Let $\mathscr{S} = \mathscr{S}(X, R^p)$ denote the collection of closed syndetic subgroups of R^p which are self-enveloping for $(X, \boldsymbol{R}^p)$. It is obvious that $(X, \boldsymbol{R}^p)$ is totally minimal if and only if $\mathscr{S} = \{\boldsymbol{R}^p\}$.