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BMO FUNCTIONS AND THE $\bar{\partial}$-EQUATION

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The $\bar{\partial}$-equation associated with the Corona problem for several complex variables is examined and the relation of that equation with BMO functions on the boundary is brought to light. A new characterisation, closely related with the H^1 duality, for BMO functions is obtained.

0. **Introduction.** This paper came out of an unsuccessful attempt to prove the Corona theorem for n-dimensions.

If we try to generalise L. Carleson's 1-dimensional proof (with the modifications introduced by L. Hörmander) (cf. [1], [2], [9]), we come up against the following problem:

Solve the $\bar{\partial}$-equation

$$\bar{\partial}u = \mu$$

in, say, the complex n-ball where μ is an arbitrary $\bar{\partial}$-closed differential form that satisfies an appropriate Carleson condition and where we require the solution u to have L^∞ boundary values (also in an appropriate sense, cf. [9]).

We shall show in Part 3 of this paper that it is not always possible to solve the above equation, and that the best we can obtain in general for the boundary values of the solution is a BMO condition.

However along the way a number of positive results will be obtained. In Part 1 we obtain a new characterisation of BMO functions which is closely related with the BMO, H^1 duality. This characterisation, grosso motto, runs as follows: $f \in L^1(\boldsymbol{R}^n)$ is a BMO function in $\boldsymbol{R}^n$ if and only if it is the boundary value of some function F defined in the upper half space $\boldsymbol{R}_+^{n+1}$ such that

$$|\nabla F| = \left(\sum_{i=1}^{n}\left|\frac{\partial F}{\partial x_i}\right| + \left|\frac{\partial F}{\partial y}\right|\right)d(\mathrm{Vol})$$

is a Carleson measure. Exact statements will be given later. The extension F of f in the upper half space is *not*, in general, the harmonic extension and it is not easy to describe it explicitly.

In Part 2 the above results are generalised to the complex ball and to general strictly pseudoconvex domains. This generalisation is tedious but essentially routine.

In Part 3 the real "raison d'etre" of this characterisation appears and it is used to study the $\bar{\partial}$-equation and the Corona problem. It