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GENERATORS OF FACTORS OF BERNOULLI SHIFTS

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One of the questions of ergodic theory is that of "relative position" of factors of Bernoulli shift. If $\mathscr{F}_0$ and $\mathscr{F}_1$ are factor algebras for a Bernoulli shift T, under what conditions is there an isomorphism ϕ commuting with T such that $\phi\mathscr{F}_0 = \mathscr{F}_1$?

In this paper, we give an example of a Bernoulli shift T of a space X and uncountably many partitions $\{Q_\alpha: \alpha \in A\}$ of X with the properties:

(1) $(T, Q_\alpha) \cong (T, Q_\beta)$ for $\alpha, \beta \in A$.

(2) $\bigvee_{-\infty}^{\infty} T^i Q_\alpha$ is maximal for its entropy whenever $\alpha \in A$.

(3) There is no isomorphism ϕ commuting with T such that $\phi Q_\alpha = Q_\beta$ unless $\alpha = \beta$.

If T is an automorphism of a probability space $(X, \mathscr{F}, \mu)$, a sub-sigma algebra $\mathscr{F}_0$ of $\mathscr{F}$ is a *factor algebra* for T if T is an automorphism of $(X, \mathscr{F}_0, \mu)$. That is, $\mathscr{F}_0$ is a factor algebra for T means $A \in \mathscr{F}_0 \Leftrightarrow TA \in \mathscr{F}_0$. If $\mathscr{F}_0$ is a factor algebra for T, the automorphism T restricted to $(X, \mathscr{F}_0, \mu)$ is called a *factor* of T, denoted $T|\mathscr{F}_0$.

It is clear that factors of ergodic, weakly mixing, mixing, or Kolmogorov automorphisms are ergodic weakly mixing, mixing, or Kolmogorov respectively. It is known (Ornstein) that factors of Bernoulli shifts are Bernoulli. We investigate the "relative position" of factors of Bernoulli shifts.

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Some of the questions one might ask about factors of Bernoulli shifts are:

(1) If T and T' are isomorphic Bernoulli shifts (on spaces X and X') with respective factor algebras $\mathscr{F}_0$ and $\mathscr{F}_0'$, under what conditions is there an isomorphism ϕ (defined except on a set of measure zero) such that

$$\begin{array}{ccc} X & \xrightarrow{\phi} & X' \\ {\scriptstyle T}\downarrow & & \downarrow{\scriptstyle T'} \\ X & \xrightarrow[\phi]{} & X' \end{array} \quad \text{commutes}$$