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CONSTRUCTING NEW R-SEQUENCES

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R-sequences play an important role in modern commutative algebra. The purpose of this paper is to show how new R-sequences may be constructed from a given one. In the first section we give some general results, which are applied in the second section to obtain an explicit method of construction.

Recall that a sequence of elements $x_1, \cdots, x_n$ in R is an R-sequence if $(x_1, \cdots, x_n)R \neq R$, x_1 is a nonzero divisor on R, and for $2 \leqq i \leqq n$, x_i is a nonzero divisor on $R/(x_1, \cdots, x_{i-1})R$.

Throughout this paper R will be a commutative noetherian ring which contains a field K. Moreover, R will either be local or graded.

I wish to thank Melvin Hochster for showing me Proposition 1.5, which simplified this paper considerably.

1. It is easy to see that if $x_1, \cdots, x_n \varepsilon R$ and $X_1, \cdots, X_n$ are independent indeterminates over K, and if $\varphi: K[X_1, \cdots, X_n] \rightarrow R$ by $\varphi(f(X_1, \cdots, X_n)) = f(x_1, \cdots, x_n)$ is a flat monomorphism, then $x_1, \cdots, x_n$ is an R-sequence. The converse, when R is local, is due to Hartshorne [3].

PROPOSITION 1.1 (Hartshorne). *Suppose R is local. If $x_1, \cdots, x_n \varepsilon R$ form an R-sequence then $\varphi: K[X_1, \cdots, X_n] \rightarrow R$ is a flat monomorphism, where φ is the map determined by $\varphi(X_i) = x_i$ for each i and $\varphi(a) = a$ for all $a \in K$.*

REMARK. Saying that φ is a monomorphism is the same as saying that $x_1, \cdots, x_n$ are algebraically independent over K.

COROLLARY 1.2. *Assume R is local. Suppose $f_1, \cdots, f_n$ is a $K[X_1, \cdots, X_n]$-sequence, and each $f_i \in (X_1, \cdots, X_n)K[X_1, \cdots, X_n]$. Suppose also that $x_1, \cdots, x_n$ is an R-sequence. Then*

$$f_1(x_1, \cdots, x_n), \cdots, f_n(x_1, \cdots, x_n)$$

is an R-sequence.

Proof. By Proposition 1.1 the map φ is a flat monomorphism. By flatness, since $f_1, \cdots, f_n$ is a $K[X_1, \cdots, X_n]$-sequence, $\varphi(f_1), \cdots, \varphi(f_n)$