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# MAXIMAL SUBMONOIDS OF THE TRANSLATIONAL HULL

MARIO PETRICH

**Maximal submonoids of a semigroup have recently attracted attention in semigroup literature. This is particularly true for the semigroup $\mathscr{B}(X)$ of binary relations on a set. The interesting results of Zareckiĭ in this direction point to the fact that some of these statements pertain to the more general situation of the translational hull of a Rees matrix semigroup. More generally, we consider here maximal submonoids of the translational hull of a regular semigroup.**

The first, and the main, theorem in this paper says that if $\omega$ is an idempotent bitranslation of a regular semigroup $S$, then $\omega\Omega(S)\omega \cong \Omega(\omega S\omega)$; here $\omega\Omega(S)\omega$ is a maximal submonoid of $\Omega(S)$. The second theorem pertains to subdirect irreducibility of certain subsemigroups of the translational hull of a Rees matrix semigroup. Finally, the third theorem concerns regular semigroups in which every maximal submonoid is a retract. These results have a number of consequences. The paper ends with several examples of concrete semigroups to which some of the preceding results are applied.

We start with a list of needed definitions and simple results. Let $S$ be a semigroup. A function $\lambda$ (resp. $\rho$), written on the left (resp. right) is a *left* (resp. *right*) *translation* of $S$ if $\lambda(xy) = (\lambda x)y$ (resp. $(xy)\rho = x(y\rho)$) for all $x, y \in S$. The set $\Lambda(S)$ (resp. $P(S)$) of all left (resp. right) translations of $S$ under composition $(\lambda\lambda)'x = \lambda(\lambda'z)$ (resp. $x(\rho\rho') = (x\rho)\rho'$) is a semigroup. The pair $(\lambda, \rho) \in \Lambda(S) \times P(S)$ is a *bitranslation* of $S$ if $x(\lambda y) = (x\rho)y$ for all $x, y \in S$; the subsemigroup of $\Lambda(S) \times P(S)$ consisting of all bitranslations is the *translational hull* $\Omega(S)$ of $S$. Its elements will be usually written as $\omega = (\lambda, \rho)$, where $\omega$ is considered as a bioperator on $S$. For any $s \in S$, the function $\lambda_s$ (resp. $\rho_s$) defined by $\lambda_s = sx$ (resp. $x\rho_s = xs$) for all $x \in S$, is the *inner left* (resp. *right*) *translation* and $\pi_s = (\lambda_s, \rho_s)$ is the *inner bitranslation* of $S$ induced by $s$. The set $\Pi(S) = \{\pi_s | s \in S\}$ is an ideal of $\Omega(S)$ called its *inner part*. The mapping $\pi: s \to \pi_s$ is the *canonical homomorphism* of $S$ into $\Omega(S)$. It is one-to-one if and only if $S$ is *weakly reductive*. In such a case for any $(\lambda, \rho), (\lambda', \rho') \in \Omega(S)$, $s \in S$, we have $(\lambda s)\rho = \lambda(s\rho)$, and thus all parentheses may be omitted.

An element $s \in S$ is *regular* if $s = sts$ for some $t \in S$; if also $t = tst$, then $t$ is an *inverse* of $s$. A semigroup in which every element is regular is a *regular semigroup*. Note that every regular