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ON LOOP SPACES WITHOUT p TORSION II

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Let (X, μ) be a 1-connected H-space such that $H^*(\Omega X; Q_p)$ is torsion free. We study the torsion in $H^*(X; Q_p)$ as well as its algebra structure. In particular we characterize lack of torsion in $H^*(\Omega X; Q_p)$ in terms of the module of indecomposables $Q(H^*(X; Q_p))$. We also study the Steenrod module structure of $Q(H^*(X; Z_p))$.

1. Introduction. In this paper we will study 1-connected H-spaces (X, μ) which have the homotopy type of a CW complex of finite type. Let p be a prime and Q_p be the integers localized at the prime p. Let ΩX be the loop space of X. We will assume that $H^*(\Omega X; Q_p)$ is torsion free and study the consequences for $H^*(X; Q_p)$. Our main results generalize those established in [5] where we worked with the stronger hypothesis that X has the homotopy type of a finite CW complex. We will assume familiarity with [5].

Let T be the torsion subgroup of $H^*(X; Q_p)$. It is an ideal of $H^*(X; Q_p)$. Let $F = H^*(X; Q_p)/T$. We first prove

THEOREM 1.1. *Let (X, μ) be a 1-connecsed H-space such that $H^*(\Omega X; Q_p)$ is torsion free. Then $H^*(X; Q_p)$ has no higher p torsion and F is a free commutative algebra.*

The arguments used in establishing 1.1 enable us to characterize lack of p torsion in X in terms of the cohomology of X. Let Z_p be the integers reduced mod p. Let $\rho: Q_p \to Z_p$ be the reduction mod p map. We will also use ρ to denote the induced cohomology map $\rho: H^*(X; Q_p) \to H^*(X; Z_p)$. The action of the Steenrod algebra $A^*(p)$ on $H^*(X; Z_p)$ induces a Steenrod module structure on $Q(H^*(X; Z_p))$. Let $K = \sum_{m \geq 1} \beta_p \mathscr{P}^m Q(H^{2m+1}(X; Z_p))$. Note that for $p = 2$, K is trivial. Let $\bar{Q} = Q(H^{\text{even}}(X; Z_p))/K$. The map ρ induces a map $\alpha: Q(H^{\text{even}}(X; Q_p)) \to \bar{Q}$. The quotient map $H^*(X; Q_p) \to F$ induces a map $\beta: Q(H^*(X; Q_p)) \to Q(F)$.

THEOREM 1.2. *Let (X, μ) be a 1-connected H-space. Then $H^*(QX; Q_p)$ is torsion free if, and only if, $Q(H^*(X; Q_p))$ contains a torsion free submodule M such that:*

(a) *α induces an isomorphism $M \otimes Z_p \cong \bar{Q}$*

(b) *β induces an isomorphism $M \cong Q^{\text{even}}(F)$.*

As in [5] one of the principal tools used to prove results such