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CONCERNING PARTIAL RECURSIVE SIMILARITY TRANSFORMATIONS OF LINEARLY ORDERED SETS

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Interesting contrasts between uncountable suborderings of the continuum and denumerable linear orderings are provided by results of Dushnik and Miller and Sierpiński on the one hand and Laver on other. We investigate analogues of these results in a recursive setting where the only similarity maps are restrictions of partial recursive functions. Complements of recursively enumerable bi-dense subsets of the rationals of arbitrary nonzero degree of unsolvability are shown to bear a strong resemblance to uncountable suborderings of the continuum.

1. Introduction and summary. Two linear orderings, H and G, are said to be *similar* if there is an order preserving map from H onto G. H is said to be *embeddable* in G if H is similar to a subordering of G. $H \leqq G$ denotes that H is embeddable in G while $H \nleqq G$ denotes that H is not embeddable in G. $\mathbf{c}$ denotes the cardinality of the continuum C. $\aleph$ denotes the least ordinal whose cardinality is $\mathbf{c}$. A is said to be *bi-dense* in B if both A and its complement in B are dense in B.

Theorems I and II below are due to Dushnik and Miller [1940]. Sierpiński [1950] contains Theorems III, IV, and V, whose proofs are interesting elaborations of the techniques and proofs of Dushnik and Miller.

THEOREM I. *Every countable linear ordering is similar to proper subset of itself.*

THEOREM II. *There is a bi-dense subset A of C of cardinality $\mathbf{c}$ such that there is no order-preserving map from A into itself except the identity; in particular, A is not similar to any proper subset of itself.*

THEOREM III. *There is a sequence $\{G_\alpha \mid \alpha < \aleph\}$ of bi-dense subsets of C such that each G_α has cardinality $\mathbf{c}$ and $\alpha < \beta < \aleph$ implies $G_\alpha \leqq G_\beta$ but $G_\beta \nleqq G_\alpha$.*

THEOREM IV. *There is a sequence $\{D_\alpha \mid \alpha < \aleph\}$ of bi-dense subsets*