PACIFIC JOURNAL OF MATHEMATICS
Vol. 71, No. 2, 1977

TOTAL POSITIVITY AND THE EXACT n-WIDTH OF CERTAIN SETS IN L^1

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In this paper we obtain the exact value of the L^1 n-width, both in the sense of Kolmogorov and Gel'fand, and characterize optimal subspaces for the set

$$\mathscr{K}_r = \left\{\sum_{j=1}^{r} a_j k_j(t) + \int_0^1 K(t, s)h(s)ds \colon (a_1, \cdots, a_r) \in \boldsymbol{R}^r,\ \|h\|_1 \leqq 1\right\},$$

under certain total positivity assumptions on

$$\{k_1(t), \cdots, k_r(t), K(t, s)\}.$$

A matrix analogue is also described.

1. Introduction. Let X be a normed linear space, $\mathscr{A}$ a subset of X, and X_n any n-dimensional linear subspace of X. Then the n-width of $\mathscr{A}$ relative to X, in the sense of Kolmogorov, is defined to be

$$d_n(\mathscr{A}; X) = \inf_{X_n} \sup_{x \in \mathscr{A}} \inf_{y \in X_n} \|x - y\|.$$

X_n is called an optimal subspace for $\mathscr{A}$ provided that

$$d_n(\mathscr{A}; X) = \delta(\mathscr{A}; X_n) = \sup_{x \in \mathscr{A}} \inf_{y \in X_n} \|x - y\|.$$

The n-width of $\mathscr{A}$ relative to X, in the sense of Gel'fand, is defined as

$$d^n(\mathscr{A}; X) = \inf_{L_n} \sup_{x \in \mathscr{A} \cap L_n} \|x\|,$$

where L_n is any subspace of X of codimension n. If

$$d^n(\mathscr{A}; X) = \sup_{x \in \mathscr{A} \cap L_n} \|x\|,$$

then L_n is an optimal subspace for the Gel'fand n-width of $\mathscr{A}$.

A typical choice for $\mathscr{A}$ is the image of the unit ball under a compact mapping K of X into itself,

$$\mathscr{K} = \{Kx \colon \|x\| \leqq 1\}.$$

When X is a Hilbert space then it is possible to obtain an exact value for $d_n(\mathscr{K}; X)$. This fact originated with the methods used in Kolmogorov's seminal paper [4]. For $X = L^\infty[0, 1]$, we computed (in [6]) the n-widths of $\mathscr{K}$ when K is an integral operator determined by a totally positive kernel.

In this paper, we obtain the exact value of the L^1 n-width, both