

## $T^n$ -ACTIONS ON SIMPLY CONNECTED $(n + 2)$ -MANIFOLDS

DENNIS MCGAVRAN

**In this paper we show that, for each  $n \geq 2$ , there is a unique, closed, compact, connected, simply connected  $(n + 2)$ -manifold,  $M_{n+2}$ , admitting an action of  $T^n$  satisfying the following condition: there are exactly  $n$   $T^1$ -stability groups  $T_1, \dots, T_n$  with each  $F(T_i, M_{n+2})$  connected. In this case we have  $T^n \cong T_1 \times \dots \times T_n$ . Any other action  $(T^n, M^{n+2})$ ,  $M^{n+2}$  simply connected, can be obtained from an action  $(T^n, M_{n+2})$  by equivariantly replacing copies of  $D^4 \times T^{n-2}$  with copies of  $S^3 \times D^2 \times T^{n-3}$ . As an application, we classify all actions of  $T^n$  on simply connected  $(n + 2)$ -manifolds for  $n = 3, 4$ .**

Several results have been obtained about  $T^n$ -actions on  $(n + 2)$ -manifolds. Orlik and Raymond have obtained various classification theorems for the cases  $n = 1, 2$  (see [11], [12] and [14]). Various general results have been obtained in [4] and [5] for  $n > 2$ . This paper is a continuation of the work done in [4]. We also obtain classification theorems similar to those of [12] for  $n = 3, 4$ .

In [4] it was shown that, for each  $n$ , there exist actions of  $T^n$  on simply connected  $(n + 2)$ -manifolds. Here we prove the following.

**THEOREM.** *For each  $n$ , there is a unique closed, compact, connected, simply connected  $(n + 2)$ -manifold  $M_{n+2}$  admitting an action of  $T^n$  satisfying the following conditions:*

- (i) *There are exactly  $n$   $T^1$ -stability groups  $T_1, \dots, T_n$ .*
- (ii) *Each  $F(T_i, M_{n+2})$  is connected.*

*Furthermore,  $T^n \cong T_1 \times \dots \times T_n$ .*

We then show that any action  $(T^n, M^{n+2})$ ,  $M^{n+2}$  a closed, compact, connected, simply connected  $(n + 2)$ -manifold, can be obtained from an action  $(T^n, M_{n+2})$  by equivariantly replacing copies of  $D^4 \times T^{n-2}$  with copies of  $S^3 \times D^2 \times T^{n-3}$ .

The above results are applied to two specific cases. We show that if  $T^3$  acts on a simply connected 5-manifold,  $M$ , then  $M$  is  $M_5 = S^5$  or a connected sum of copies of  $S^2 \times S^3$ . For  $T^4$ -actions on simply connected 6-manifolds,  $M$ , we show that  $M$  is  $M_6 = S^3 \times S^3$  or  $M$  is a connected sum of copies of  $S^2 \times S^4$  and  $S^3 \times S^3$ .

**1. Preliminaries.** We shall use standard terminology and notation throughout (e.g. see [2]). Unless otherwise stated, all mani-