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THE MAXIMAL RIGHT QUOTIENT SEMIGROUP OF A STRONG SEMILATTICE OF SEMIGROUPS

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Let S be a strong semilattice Y of monoids. If S is right nonsingular then Y is nonsingular. The converse is true when S is a sturdy semilattice Y of right cancellative monoids. Should S have trivial multiplication then each monoid of more than one element has as its index an atom of Y. Finally, if S is a right nonsingular strong semilattice Y of principal right ideal Ore monoids with onto linking homomorphisms then $Q(S)$, the maximal right quotient semigroup of S, is a semilattice $Q(Y)$ of groups.

1. **Introduction.** Let Y be a semilattice and let $\{S_\alpha\}_{\alpha \in Y}$ be a collection of pairwise disjoint semigroups. For each pair α, $\beta \in Y$ with $\alpha \geqq \beta$, let $\psi_{\alpha,\beta}\colon S_\alpha \rightarrow S_\beta$ be a semigroup homomorphism such that $\psi_{\alpha,\alpha}$ is the identity mapping and if $\alpha > \beta > \gamma$ then $\psi_{\alpha,\gamma} = \psi_{\beta,\gamma}\psi_{\alpha,\beta}$. Let $S = \bigcup_{\alpha \in Y} S_\alpha$ with multiplication

$$a*b = \psi_{\alpha,\alpha\beta}(a)\psi_{\beta,\alpha\beta}(b)$$

for $a \in S_\alpha$ and $b \in S_\beta$. The semigroup S is called a *strong semilattice Y of semigroups S_α*. If, in addition, each $\psi_{\alpha,\beta}$ is one-to-one then S is called a *sturdy semilattice of semigroups*. The basic terminology in use throughout this paper can be found in [1], [7], and [9]. Note that a semilattice of groups [1, p. 128] is a strong semilattice of semigroups. In [6], McMorris showed that if M is a semilattice X of groups G_δ, then $Q(M)$, the maximal right quotient semigroup of M, is also a semilattice of groups. Hinkle [2] constructed $Q(M)$ and showed that its indexing semilattice is $Q(X)$.

Let S be a semigroup with 0. A right ideal D of S is *dense* if for each s_1, s_2, $s \in S$ with $s_1 \neq s_2$, there exists an element $d \in D$ such that $s_1 d \neq s_2 d$ and $sd \in D$. A right ideal L of S is *$\cap$-large* if for each nonzero right ideal R of S, $R \cap L \neq \{0\}$. It is easy to see that dense implies $\cap$-large. If each $\cap$-large right ideal of S is also dense then S is said to be *right nonsingular*. If a semigroup is commutative or each one-sided ideal is two-sided then we will use the term nonsingular. Let T be a right S-system with 0[5] then the *singular congruence* ψ_T on T is a right congruence defined for a, $b \in T$ by $a\psi_T b$ if and only if $as = bs$ for all s in an $\cap$-large right ideal of S. McMorris [8] showed that $\psi_S = i_S$, the identity congruence on S, if and only if S is right nonsingular.

Recently it has been shown [4], [5] that if S is a commutative