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INVERSE LIMITS AND MAPPINGS OF MINIMAL TOPOLOGICAL SPACES

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S. W. Willard has conjectured that every H-closed space is the continuous image of a minimal Hausdorff space. In this paper we verify Willard's conjecture and show as well that every R-closed space is the continuous image of a minimal regular space. We also identify conditions sufficient to guarantee that an H-closed space be the finite-to-one continuous image of a minimal Hausdorff space. We give an example of a nonvacuously $R(ii)$ space whose product with itself is neither $R(i)$ nor $R(ii)$, and we obtain a number of results concerning inverse limits of H-closed spaces and R-closed spaces.

1. Introduction. Throughout this paper, the word *map* (or *mapping*) will always mean a continuous function.

If P is a topological property, then a P-space is called *P-closed* if it is closed in every P-space in which it is embedded and *minimal* P if there is no strictly coarser P topology on the same underlying set. For $P =$ Hausdorff [$P =$ regular T_1] the P-closed and minimal P properties will be denoted as H-closed [R-closed] and MH [MR]. In studying mapping properties of MH spaces, S. W. Willard [14] showed that the Hausdorff spaces whose Hausdorff continuous images are always MH are precisely the functionally compact spaces of Dickman and Zame [4] and conjectured that the Hausdorff spaces which are the continuous images of MH spaces are precisely the H-closed spaces. An analogous conjecture can be posed for MR and R-closed spaces. Here we shall prove that every H-closed [R-closed] space is the image, under an open and perfect mapping, of an MH [MR] space of the same weight. Since every Hausdorff [regular T_1] continuous image of an H-closed [R-closed] space is H-closed [R-closed] (see [2]), we thereby establish both of the above mentioned conjectures. We also obtain results concerning products of MR spaces and products of $R(ii)$ spaces (see §2 for definition) as well as a number of theorems concerning inverse limits of H-closed spaces and of R-closed spaces. In §4 we determine conditions that guarantee that an H-closed space be the image of an MH space under an at-most-two-to-one mapping.

A set V in a topological space is *regularly open* if $V = \text{int}\, \bar{V}$. A space is *semiregular at a point* if that point has a neighborhood base of regularly open sets. A point of a topological space at which the space is semiregular will be called a *semiregular point* of the