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STABLE ISOMORPHISM AND STRONG MORITA EQUIVALENCE OF C^*-ALGEBRAS

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We show that if A and B are C^*-algebras which possess countable approximate identities, then A and B are stably isomorphic if and only if they are strongly Morita equivalent. By considering Breuer ideals, we show that this may fail in the absence of countable approximate identities. Finally we discuss the Picard groups of C^*-algebras, especially for stable algebras.

0. **Introduction.** Theorem 2.8 of [4] states that if B is a full hereditary subalgebra of a C^*-algebra A, and if each of A and B have strictly positive elements (or, equivalently, countable approximate identities, by [1]), then B is stably isomorphic to A, that is, $B \otimes K$ is isomorphic to $A \otimes K$ where K is the algebra of compact operators on a separable infinite dimensional Hilbert space. The purpose of the present paper is to show how the above theorem implies that two C^*-algebras which are strongly Morita equivalent in the sense of having an imprimitivity bimodule [8, 9, 10], will be stably isomorphic if they possess strictly positive elements, and in particular if they are separable. (The converse is readily apparent.) On the other hand, in the second section, by considering Breuer ideals, we give examples of pairs of C^*-algebras which are strongly Morita equivalent but are not stably isomorphic, even if we allow tensor products with $K(H)$ for nonseparable H. (Of course, one of them will fail to have a strictly positive element.) Finally, in the last section we discuss the Picard groups of C^*-algebras. We show in particular that the Picard group of any C^*-algebra, B, which is stable (that is, $B \cong B \otimes K$) and has a strictly positive element, is isomorphic to the quotient of the automorphism group of B by the subgroup of generalized inner automorphisms of B.

1. **The main theorem.** Let A be a C^*-algebra, and let $M(A)$ denote the double centralizer algebra of A. By a corner of A we mean [4] a subalgebra of the form pAp where p is a projection in $M(A)$. A corner is said to be full if it is not contained in any proper two-sided ideal of A, that is, if ApA is dense in A. Two corners, pAp and qAq, are called *complementary* if $p + q = 1$. The device by which we relate strong Morita equivalence to the setting of [4] is: