PACIFIC JOURNAL OF MATHEMATICS
Vol. 71, No. 2, 1977

STABLE ISOMORPHISM OF HEREDITARY SUBALGEBRAS OF C^*-ALGEBRAS

LAWRENCE G. BROWN

The main theorem is that if B is a hereditary C^*-subalgebra of A which is not contained in any proper closed two-sided ideal, then under a suitable separability hypothesis $A \otimes \mathscr{K}$ is isomorphic to $B \otimes \mathscr{K}$, where $\mathscr{K}$ is the C^*-algebra of compact operators on a separable infinite-dimensional Hilbert space. In the special case where $A = C \otimes \mathscr{K}$ and $B = pAp$ for some projection p in the double centralizer algebra, $M(A)$, of A such that p commutes with $C \otimes 1 \subset M(A)$, the theorem follows from a result of Dixmier and Douady [12]. In fact p must be defined by a continuous mapping from the spectrum $\hat{C}$ of C to the strong grassmanian. Thus p defines a continuous field of Hlibert spaces on $\hat{C}$ and [12] implies that the countable direct sum of this field with itself is trivial. Our proof amounts to an abstraction of [12]. The theorem also leads to an abstraction and generalization of some results of Douglas, Fillmore, and us on extensions of C^*-algebras ([6, § 3]). The final section of the paper contains a generalization of the Dauns-Hofmann theorem which is needed to justify some of our remarks.

1. Preliminaries. If B is a C^*-subalgebra of A, B is called *hereditary* if $a \in A$, $b \in B$, $0 \leqq a \leqq b$ imply $a \in B$. B is *full* if it is not contained in any proper closed two-sided ideal of A.

We will be particularly concerned with hereditary subalgebras which are related to the double centralizer algebra, $M(A)$, of A. An element of $M(A)$ is a pair $x = (S, T)$, where S and T are linear operators on A such that $a \cdot S(b) = T(a) \cdot b$ for all $a, b \in A$. $M(A)$ is the universal C^*-algebra containing A as a two-sided ideal. S and T are the restrictions to A of the left and right multiplications by x. The *strict topology* of $M(A)$ is the weakest topology in which the maps $x \to xa$ and $a \to ax$ are continuous for each $a \in A$, where A has the norm topology. Any nondegenerate representation, π, of A extends uniquely to a representation, $\tilde{\pi}$, of $M(A)$ on the same Hilbert space. For further details on $M(A)$ the reader is referred to [8] and [3].

If p is a projection in $M(A)$, pAp is a hereditary subalgebra of A which will be called a *corner*.

LEMMA 1.1. *If p is a projection in $M(A)$, the following are equivalent:*

1. *For any nondegenerate representation, π, of A, $\tilde{\pi}(p) \neq 0$.*