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PROJECTIVE IDEALS IN RINGS OF CONTINUOUS FUNCTIONS

J. GLENN BROOKSHEAR

An ideal in a ring Λ is said to be projective provided it is a projective Λ-module. This paper is concerned with the problem of topologically characterizing projectivity within the class of ideals of a ring of continuous functions. Since there are projective and nonprojective ideals having the same z-filter, the possibility of such a characterization appears remote. However, such a characterization is shown to exist for the projective z-ideals. Moreover, a relationship between projective z-ideals and arbitrary projective ideals is exhibited and used to show that, in some cases, every projective ideal is module isomorphic to a projective z-ideal.

1. **Preliminaries.** Let X be a completely regular, Hausdorff space and $C(X)$ be the ring of real-valued continuous functions on X. An ideal in $C(X)$ is said to be projective provided it is a projective $C(X)$-module. In [1], Bkouche has shown that if X is locally compact then $C_K(X)$, the ideal of functions with compact support, is projective if and only if X is paracompact. Actually, he has characterized projectivity within the class of pure submodules of $C(X)$ in terms of the topological properties of βX, the Stone-Čech compactification of X. Using the concept of a projective basis, Finney and Rotman [5] have presented a direct proof of Bkouche's result for locally compact spaces. This paper is concerned with the problem of topologically characterizing projectivity within the class of all ideals in $C(X)$.

The remaining paragraphs in this section introduce the terminology and notation which is used in the sequel. The reader is referred to [6] for additional background. In § 2 a characterization of projectivity in the class of ideals in $C(X)$ is given which is used to show the existence of projective and nonprojective ideals having the same z-filter. Such examples indicate that the topology of a space is not rich enough to distinguish between the projective and nonprojective ideals in the general setting. In § 3 projectivity within the class of z-ideals is topologically characterized and these results are shown to be a generalization of the work of Bkouche. In § 4 the general problem is again addressed. Here it is shown that any projective ideal I is closely associated with a projective z-ideal I_z. The relationship between I and I_z is studied and it is shown that often I is module isomorphic to I_z. Hence, in some