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A CHARACTERIZATION OF $PSp(2m, q)$ AND $P\Omega(2m+1, q)$ AS RANK 3 PERMUTATION GROUPS

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This paper characterizes the projective symplectic groups $PSp(2m, q)$ and the projective orthogonal groups $P\Omega(2m+1, q)$ as the only transitive rank 3 permutation groups G of a set X for which the pointwise stabilizer of G has orbit lengths 1, $q(q^{2m-2}-1)/(q-1)$ and q^{2m-1} under a relatively weak hypothesis about the pointwise stabilizer of a certain subset of X. A precise statement is

THEOREM. **Let G be a transitive rank 3 group of permutations of a set X such that the orbit lengths for the pointwise stabilizer are 1, $q(q^{r-2}-1)/(q-1)$ and q^{r-1} for integers $q>1$ and $r>4$. Let $x^{\perp}$ denote the union of the orbits of length 1 and $q(q^{r-2}-1)/(q-1)$. Let $R(xy)$ denote $\cap\{z^{\perp}: x, y \in z^{\perp}\}$. Assume $R(xy) \neq \{x, y\}$ for $y \in x^{\perp}-\{x\}$. Assume that the pointwise stabilizer of $x^{\perp} \cap y^{\perp}$ for $y \notin x^{\perp}$ does not fix $R(xy)$ pointwise. Then r is even, q is a prime power and G is isomorphic to either a group of symplectic collineations of projective $(r-1)$ space over $GF(q)$ containing $PSp(r, q)$ or a group of orthogonal collineations of projective r space over $GF(q)$ containing $P\Omega(r+1, q)$.**

1. Introduction.

The projective classical groups of symplectic type $PSp(2m, q)$ for $m \geqq 2$ are transitive permutation groups of rank 3 when considered as groups of permutations of the absolute points of the corresponding projective space. Indeed the pointwise stabilizer of $PSp(2m, q)$ has 3 orbits of lengths 1, $q(q^{2m-2}-1)/(q-1)$ and q^{2m-1}. In a recent paper [7], the author characterized the symplectic groups $PSp(2m, q)$ for $m \geqq 3$ as rank 3 permutation groups.

THEOREM A. *Let G be a transitive rank 3 group of permutations of a set X such that G_x, the stabilizer of a point $x \in X$, has orbit lengths 1, $q(q^{r-2}-1)/(q-1)$ and q^{r-1} for integers $q \geqq 2$ and $r \geqq 5$. Let $x^{\perp}$ denote the union of the G_x-orbits of lengths 1 and $q(q^{r-2}-1)/(q-1)$. Let $R(xy)$ denote $\cap\{z^{\perp}: x, y \in z^{\perp}\}$. Assume $R(xy) \neq \{x, y\}$. Assume that the pointwise stabilizer of $x^{\perp}$ is transitive on the points unequal to x of $R(xy)$ for $y \notin x^{\perp}$. Then r is even, q is a prime power and G is isomorphic to a group of symplectic collineations of projective $(r-1)$ space over the field of q elements, which contains $PSp(r, q)$.*