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THE SECOND DUAL OF $C(X)$

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In this paper, we undertake a study of the order dual, denoted M, of the radon measures of compact support on a locally compact space X. In the case that X is realcompact, M is the second (order) dual of the space of continuous functions on X, $C(X)$. We define the sublattice of semi-continuous elements, $S(X)$, and prove that each member of M is dominated by a member of $S(X)$. It follows that the ideal generated by $S(X)$ in M is all of M. On the other hand, the ideal generated by $C(X)$ in M is all of M if and only if X is a cb-space.

Finally, we show that $S(X)$ and $C(X)$ can be identified in M as certain spaces of multiplication operators which are continuous with respect to certain weak topologies. This extends the work of J. Mack, who first characterized M as the (continuous) multiplication operators on the Radon measures.

Introduction. In [3] Kaplan considered $C_k(X) = C_k$, the continuous functions of compact support on a locally compact space, and its order dual L_k (the space of Radon measures). In the process, he singled out $\cup L(K)$, the ideal of those measures having compact support. It is the order dual of this space, denoted M, in which we will be interested. In the case that X is realcompact, M is the second dual of the space of continuous functions and therefore of particular interest.

M has already been studied by Mack [5], who characterized it as the set of (order) continuous multiplication operators on L_k. It is our purpose to extend his work. In considering the case where X is compact, Kaplan studied various sublattices of M including what he called the semi-continuous elements $S(X)$. We will extend the study to our more general setting and show that $S(X)$ and $C(X)$ can be identified in M as spaces of multiplication operators on L_k, continuous with respect to certain weak topologies. Thus we will relate the work of the two authors.

1. Preliminaries. The information and results summarized here will be used frequently in the rest of the paper. We assume a knowledge of the basic results on Riesz spaces.

1.1. A subset B of a Riesz space (vector lattice) E is called