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APPROXIMATELY DIFFERENTIABLE FUNCTIONS: THE r TOPOLOGY

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It is shown that the coarsest topology making all approximately differentiable functions continuous is not the density topology. The correct topology, the r topology, is introduced, and the structure of the open sets in this topology is examined. Among other things, it is proven that any r-open set must have nonempty Euclidean interior.

In the development of the r topology, two new classes of functions play a role. These classes are the Baire * 1 approximately continuous functions and the ambivalent approximately continuous functions. For either class, r is also the coarsest topology for which they are continuous.

1. Introduction. In this paper we examine functions $f: [0, 1] \to R$ which possess a finite approximate derivative everywhere in $[0, 1]$. These functions are properly contained in the class of approximately continuous functions. In their study [4] of approximately continuous transformations, Goffman and Waterman present a topology which they label the density topology d. They show that with respect to d the approximately continuous functions are continuous. For any collection of real-valued functions there is a coarsest topology relative to which each function in the collection is continuous. Goffman, Neugebauer and Nishiura [3] established that the coarsest such topology for the approximately continuous functions is precisely d. The connection between approximately differentiable functions and the density topology is clear. It presents no difficulty to show that the differentiable functions relative to d are exactly the approximately differentiable functions. Thus it would appear that the density topology is the natural tool with which to examine the approximate behavior of functions. However, in this paper an unexpected fact surfaces. The density topology is not the coarsest topology making the approximately derivable functions continuous. Here we present the proper topology which we lable the r topology. Besides being coarser than d the r topology is shown to possess several properties not common to d. For example, any set open in r must have nonempty Euclidean interior.

In the development of the r topology two new classes of functions play a role. These classes are the Baire * 1 approximately continuous functions and the ambivalent approximately continuous functions. For either class, r is also the coarsest topology for which they are continuous.