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A COMPARISON THEOREM AND OSCILLATION CRITERIA FOR SECOND ORDER DIFFERENTIAL SYSTEMS

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Let $\mathscr{H}$ be a Hilbert space and let $\mathscr{B} = \mathscr{B}(\mathscr{H}, \mathscr{H})$ be the B^*-algebra of bounded linear operators from $\mathscr{H}$ to $\mathscr{H}$ with the uniform operator topology. Let $\mathscr{S}$ be the subset of $\mathscr{B}$ consisting of the selfadjoint operators. This paper is concerned with second order, selfadjoint differential equations of the form

$$(1) \qquad [P(x)Y']' + Q(x)Y = 0$$

on $\mathscr{R}^+ = [0, \infty)$, where P and Q are continuous mappings of $\mathscr{R}^+$ into $\mathscr{S}$ with $P(x)$ positive definite for all $x \in \mathscr{R}^+$. Let $\mathscr{G}$ be the set of positive linear functionals on $\mathscr{B}$. Positive functionals are used in deriving a generalization of Sturm's comparison theorem, and, in turn, the comparison theorem is used to obtain oscillation criteria for equation (1). These criteria are shown to include a large number of well-known oscillation criteria for (1) in the matrix and scalar case. Extensions of the results to nonlinear differential equations and differential inequalities are also discussed.

Appropriate discussions of the concepts of differentiation and integration of $\mathscr{B}$-valued functions, as well as treatments of the existence and uniqueness of solutions $Y: \mathscr{R}^+ \to \mathscr{B}$ of (1), can be found in a variety of texts. See, for example, E. Hille [8, Chapters 4, 6, and 9]. Studies of the behavior of solutions of second order equations in a B^*-algebra have been done by several authors, including Hille [8, Chapter 9], T. L. Hayden and H. C. Howard [7] and C. M. Williams [18]. Of course, if $\mathscr{H} = \mathscr{R}_n$, Euclidean n-space, then $\mathscr{B}$ is the B^*-algebra of $n \times n$ matrices, and equation (1) is the familiar second order, selfadjoint matrix differential equation which has been investigated in great detail by a large number of authors. In this regard we refer to the texts by A. Coppel [2], P. Hartman [6], Hille [8], W. T. Reid [13], and C. A. Swanson [15], all of which provide comprehensive bibliographies and extensive references to the research literature.

It is easy to verify by differentiation that if $Y = Y(x)$ is a solution of equation (1), then

$$(2) \qquad Y^*(x)[P(x)Y'(x)] - [P(x)Y'(x)]^*Y(x) \equiv C$$

on $\mathscr{R}^+$, $C \in \mathscr{B}$ a constant. The solution Y is *conjoined* (or *prepared*) if the constant operator C in (2) is 0, the zero operator. The term