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APPROXIMATE FIBRATIONS AND A MOVABILITY CONDITION FOR MAPS

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In a previous paper the authors defined the approximate homotopy lifting property and studied its implications. This property is a generalization of the homotopy lifting property of classical fiber space theory. Here a necessary and sufficient condition on point-inverses for a map to have the approximate homotopy lifting property for n-cells is given; and the approximate homotopy lifting property for n-cells is shown to imply the approximate homotopy lifting property for all spaces. A corollary is that, in a fairly general context, any two point-inverses of a Serre (weak) fibration have the same shape. By combining these results with results of L. Husch, some conditions are obtained under which a map between manifolds can be approximated by locally trivial fibrations.

1. Introduction and preliminaries. Throughout this paper, $p: E \to B$ will denote a surjective map between locally compact, separable metric ANR's E and B. We say that p has the *approximate homotopy lifting property* (AHLP) *with respect to the space* X if whenever $h: X \times I \to B$ and $\tilde{h}: X \times \{0\} \to E$ are maps such that $p\tilde{h} = h | X \times \{0\}$ and ε is a cover of B, h extends to a map $\tilde{h}: X \times I \to E$ such that h and $p\tilde{h}$ are ε-close. By a simple modification of [4; XX, 2.4], if p has the AHLP with respect to X, we may choose $\tilde{h}$ to be stationary when h is, i.e., if $h(p(x), t) = p(x)$ for all t, $\tilde{h}(x, t) = x$ for all t. If p has the AHLP for all spaces, we say that p is an *approximate fibration.* (It suffices to have the AHLP for metric spaces by [3, Prop. 1.4].)

Approximate fibrations and approximate lifting were introduced in [3] as an abstraction of the useful lifting properties possessed by UV^k-maps [9], [11], [12]. It is shown in [3] that approximate fibrations have shape theoretic properties analogous to the homotopy theoretic properties of Hurewicz fibrations. For example, under appropriate hypotheses on E and B any two point inverses of p have the same shape, and p induces an exact sequence involving the homotopy groups of E and B and the shape-theoretic homotopy groups of any point inverse of p.

In this paper, we study conditions which imply that a map is an approximate fibration. Section 2 is devoted to showing that, in the case of approximate liftings, the difference between Serre and Hurewicz fibrations disappears; that is, the AHLP for all cells is