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NEW PARTIAL ASYMPTOTIC STABILITY RESULTS FOR NONLINEAR ORDINARY DIFFERENTIAL EQUATIONS

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The problem of determining sufficient conditions which that assure that all solutions of the second order equation $x'' + q(t)x = 0$ approach zero as t tends to infinity has been studied extensively since 1933. Several results have been given for generalizations of the basic linear equation. In this paper a new technique is used to obtain further results for the equation

$$(p(t)x')' + q(t)f(x) = e(t) . \tag{1}$$

The generality of the theorems developed is established by showing that a substantial number of previously known results are immediate consequences of the work herein. Of particular interest is the fact that three recent theorems by Burton and Grimmer, which appeared in this journal, follow from the work contained in this paper.

A substantial number of previously known results are immediate consequences of the work herein. In particular, it includes three recent theorems by Burton and Grimmer, which appeared in this journal, [1].

The following assumptions are made throughout the paper.

(I) The function f is continuous on $(-\infty, \infty)$ and satisfies the additional requirement that

$$xf(x) > 0 \quad \text{whenever} \quad x \neq 0 .$$

(II) The functions p and q are positive, continuous functions on $[0, \infty)$, whose product is locally of bounded variation on $[0, \infty)$. Furthermore, letting $(pq)(t) = (pq)(0) + (pq)_+(t) - (pq)_-(t)$ be the Jordan decomposition of pq, it is assumed that

$$\int_0^\infty (pq)^{-1}(\tau)d(pq)_-(\tau) < \infty .$$

(III) The function e is a locally integrable function on $[0, \infty)$ which satisfies the inequality

$$\int_0^\infty (pq)^{-1/2}(\tau)|e(\tau)|d\tau < \infty .$$