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ANOTHER NOTE ON EBERLEIN COMPACTS

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An Eberlein compact is a compact space that can be embedded in a Banach space with its weak topology. It is shown that: If X is compact and if $X=M_1\cup M_2$ with M_1 and M_2 metrizable, then $\bar{M}_1\cap\bar{M}_2$ is metrizable and X is an Eberlein compact. This answers a question of Arhangel'skiĭ.

1. Introduction. An *Eberlein compact*, or *EC*, is a compact space[1] which can be embedded in a Banach space with its weak topology. For background and various properties of these spaces, the reader is referred to [1] or the authors' preceding note [3].

Since every metrizable space can be embedded in a Banach space with its norm topology, every metrizable compact space is clearly an *EC*. The purpose of this note is to prove the following stronger result, thereby answering a question of A. V. Arhangel'skiĭ.

THEOREM 1.1. *If X is compact, and if $X = M_1\cup M_2$ with M_1 and M_2 metrizable, then $\bar{M}_1\cap\bar{M}_2$ is metrizable and X is an EC.*

In contrast to Theorem 1.1, a compact space which is the union of *three* metrizable subsets need *not* be an *EC*, or even a Fréchet space[2] (see [2, Example 6.2][3]). However, it was shown in [5] that a compact space which is the union of countably many metrizable subsets must at least be sequential (a property somewhat weaker than being a Fréchet space).

2. Proof of Theorem 1.1. We first show that $M=\bar{M}_1\cap\bar{M}_2$ is metrizable. For $i=1, 2$, let $\mathscr{U}_i$ be a σ-discrete—hence σ-disjoint—base for M_i. For each $U\in\mathscr{U}_i$, choose an open set $\phi_i(U)$ in X such that $\phi_i(U)\cap M_i=U$. Let $\mathscr{U}=\{\phi_i(U)\cap M\colon U\in\mathscr{U}_i,\ i=1, 2\}$. Then $\mathscr{U}$ is easily seen to be a σ-disjoint 1—m hence point-countable 1—m base for M. Since M is compact, it must therefore be metrizable by a result of A. S. Miščenko [4].

Since M is compact and metrizable, it has a countable base (B_n). For each pair (m, n) such that $\bar{B}_m\cap\bar{B}_n=\varnothing$, pick an open F_σ-set

[1] All spaces in this paper are Hausdorff.

[2] X is a *Fréchet* space if, whenever $x\in\bar{A}$ in X, then $x_n\to x$ for some $x_n\in A$. Every *EC* is a Fréchet space by a theorem of Eberlein and Šmulian (see [1, Theorem 4.1]).

[3] In this example, the three metrizable subsets are actually discrete, and one of them is an open set whose complement is (necessarily, by Theorem 1.1) an *EC*.