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INVARIANT SUBMODULES OF UNIMODULAR HERMITIAN FORMS

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Let M be a unimodular lattice on an indefinite hermitian space over an algebraic number field. The submodules of M invariant under the action of the special unitary group of M are classified. Generators for the local unitary groups of M are also determined.

1. Introduction. Let F be an algebraic number field of finite degree and K a quadratic extension of F. Let V be an indefinite hermitian space over K of finite dimension $n \geqq 3$ and $\Phi: V \times V \rightarrow K$ the associated nondegenerate hermitian form on V with respect to the nontrivial automorphism of K over F. Assume V supports a unimodular lattice M (in the sense of O'Meara [7; § 82G] for quadratic spaces). Denote by $U(V)$ the unitary group of V and by $U(M)$ the subgroup of isometries in $U(V)$ that leave M invariant. We will classify the sublattices of M that are invariant under the action of the special unitary group $SU(M)$. The problem is first solved locally; the global result is then obtained by applying the approximation theorem of Shimura [8; 5.12].

We now consider localization (see also [2; § 2] and [8]). Let $\mathfrak{p}$ be a finite prime spot of F and $F_{\mathfrak{p}}$ the corresponding local field. Put $K_{\mathfrak{p}} = K \otimes_F F_{\mathfrak{p}}$ and $V_{\mathfrak{p}} = V \otimes_F F_{\mathfrak{p}}$. Making the standard identifications, we have $K \subseteqq K_{\mathfrak{p}}$, $F_{\mathfrak{p}} \subseteqq K_{\mathfrak{p}}$ and $V \subseteqq V_{\mathfrak{p}}$. The hermitian form Φ on V extends naturally to an hermitian form on $V_{\mathfrak{p}}$. Let $\mathfrak{o}$ be the ring of integers in F, $\mathfrak{o}_{\mathfrak{p}}$ the (topological) closure of $\mathfrak{o}$ in $F_{\mathfrak{p}}$ and $\mathfrak{O}_{\mathfrak{p}}$ the integral closure of $\mathfrak{o}_{\mathfrak{p}}$ in $K_{\mathfrak{p}}$. Put $M_{\mathfrak{p}} = \mathfrak{O}_{\mathfrak{p}} M \subseteqq V_{\mathfrak{p}}$. Locally, we must study the submodules of $M_{\mathfrak{p}}$ invariant under the action of $SU(M_{\mathfrak{p}})$. Except when $K_{\mathfrak{p}}$ is a ramified extension of a dyadic field $F_{\mathfrak{p}}$, the classification will be trivial. For ramified dyadic extensions, it is necessary to determine a set of generators of $U(M_{\mathfrak{p}})$ before the classification can be determined.

We now state the main results.

THEOREM A. *Let M be a unimodular lattice on an indefinite hermitian space of dimension $n \geqq 3$ over an algebraic number field. Then a sublattice N of M is invariant under the action of the special unitary group $SU(M)$ if and only if for all finite prime spots $\mathfrak{p}$ of F, the localization $N_{\mathfrak{p}} = \mathfrak{O}_{\mathfrak{p}} N$ is invariant under the ac-*