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ON THE RETRACTABILITY OF SOME ONE-RELATOR GROUPS

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Recently the concept of a retractable group has been introduced. This class of groups contains the class of lattice-ordered groups as a proper subclass, and, in particular, contains the class of all torsion-free abelian groups. Retractable groups enjoy many of the properties of lattice-ordered groups; in fact, most results concerning lattice-ordered groups have immediate extensions to this wider class. In this note, we investigate the retractability of certain two-generator one-relator groups.

1. Introduction. There has been an abundance of literature on the class of groups presented by a single defining relation and, in particular, on the groups given by the presentation

$$\langle a, c \,|\, a^{-1}c^m a = c^n \rangle ,$$

where m and n are integers. In [1] the concept of a retractable group was introduced and in this note we attempt to determine which of this latter class of groups are retractable.

In Theorem 3.3 we show that the groups $\langle a, c \,|\, a^{-1}ca = c^m \rangle$, where m is a positive integer, are retractable and each admits at least a countably infinite number of retractions that satisfy condition (δ). (Definitions will be given in §§2 and 3.) It was shown in [5] that the group $\langle a, c \,|\, a^{-1}ca = c^2 \rangle$ admits exactly four full orders. Each of these induces a retraction on this group. We show in Theorem 3.5 that each of the groups $\langle a, c \,|\, a^{-1}ca = c^m \rangle$, where $m > 1$, admits exactly four lattice-orders and each of these is a full order. In Theorem 3.6 we show that the groups $\langle a, c \,|\, a^{-1}ca = c^m \rangle$, where $m < 0$, admit retractions if and only if 2 is a factor of m, and in this case, none of these groups admit lattice-orders. In Theorem 3.1 we show that if G is a retractable group and $g^n = h^n$, for some $g, h \in G$ and some natural number n, then g and h are conjugate. As a corollary to this theorem, we are able to show that the groups $\langle a, c \,|\, a^n = c^n \rangle$, where n is a natural number and $n > 1$, and

$$\langle a, c \,|\, a^{-1}c^m a = c^n \rangle ,$$

where m and n are distinct integers and $gcd(m, n) > 1$, are not retractable.