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# SQUARE INTEGRABLE REPRESENTATIONS AND THE FOURIER ALGEBRA OF A UNIMODULAR GROUP

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**Let $G$ be a unimodular group, and let $\lambda_d$ be the subrepresentation of the left regular representation $\lambda$, which is the sum of the square integrable representations. The purpose of this paper is to study the representation $\lambda_d$ with special emphasis on the closed subspace $A_d(G)$ of the Fourier algebra $A(G)$ of the group which is generated by the coefficients of $\lambda_d$. In the last part of the paper we study in detail a particular noncompact group for which $\lambda = \lambda_d$.**

We denote, as in [4], by $A(G)$ the algebra of the coefficients of $\lambda$, that is the algebra of continuous functions on $G$ of the type $(\lambda(x)f, g)$, with $f, g \in L^2(G)$. The first section contains results of a general nature: we show that $A_d(G)$ is the dual space of a $C^*$-algebra contained in $VN(G)$, the von Neumann algebra generated by the operators $\lambda(x)$, $x \in G$, and that its unit ball is the weak closure of the extreme points of the unit ball of $A(G)$. We also show that $A_d(G) \subset L^2(G)$ if and only if the formal degrees of the square integrable representations of $G$ are bounded away from zero.

In the last section we make a closer study of an example due to J. Fell of a noncompact group $G$ for which $A_d(G) = A(G)$. We show that the traces of the square integrable representations of this group are bounded measures and we construct a kind of Dirichlet kernels, which also turn out to be bounded measures.

We prove that summation with respect to these kernels converges in $L^p(G)$ for $1 < p < \infty$, but not for $L^1(G)$.

We conclude the paper with some remarks on the Wiener-Pitt phenomenon for bounded measures on this group.

1. We refer the reader to [4] for the definitions and the properties of the Fourier algebra $A(G)$ and the Fourier-Stieltjes algebra $B(G)$ of a locally compact group $G$, and to [3] for the basic facts about $C^*$-algebras, von Neumann algebras and square integrable representations of unimodular groups. Throughout the paper "group" will always mean "locally compact unimodular group" and "representation" will mean "unitary continuous representation."

Following Arsac [1], given a representation $\pi$ of $G$ on a Hilbert space $H_\pi$, we denote by $A_\pi$ the closed subspace of $B(G)$ spanned by the coefficients of $\pi$, i.e., the functions $(\pi(x)\mu | \nu)$, $x \in G$, $\mu, \nu \in H_\pi$. Let $\lambda_d$ be the subrepresentation of the left regular representation of $G$