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SEMIDIRECT PRODUCT OF SEMIGROUPS IN RELATION TO AMENABILITY, CANCELLATION PROPERTIES, AND STRONG FØLNER CONDITIONS

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The purpose of this paper is to settle two problems. The first is Sorenson's conjecture on whether every right cancellative left amenable semigroup is left cancellative. The second, posed by Argabright and Wilde, is whether every left amenable semigroup satisfies the strong Følner condition (SFC). We first show that these two problems are equivalent, then prove that the answer to both questions is no, through analyzing the semidirect product of semigroups in relation to amenability and cancellation properties. We conclude by investigating further the properties of semigroups satisfying SFC, and finally include some analogous results for left measurable semigroups.

1. Introduction. For any semigroup S, let $m(S)$ be the Banach space of bounded realvalued functions on S with the sup norm. For each $s \in S$ we define a linear operator $\ell_s[r_s]$ on $m(S)$ by $\ell_s f(t) = f(st)$ $[r_s f(t) = f(ts)]$. A mean on S is a positive element of norm one in the dual $m(S)^*$ of $m(S)$. We say that $\mu \in m(S)^*$ is left [right] invariant if $\mu(\ell_s f) = \mu(f)[\mu(r_s f) = \mu(f)]$ for each $f \in m(S)$ and $s \in S$. A semigroup is said to be left [right] amenable if it has a left [right] invariant mean, and we denote the set of left [right] invariant means on S by $M_\ell(S)[M_r(S)]$. When S is both left and right amenable we say that S is amenable. For a detailed account of the properties of left amenable semigroups, we refer to Day [2] and [3].

For any subset $A \subset S$, let χ_A denote its characteristic function, i.e., $\chi_A(s) = 1$ if $s \in A$ and $\chi_A(s) = 0$ if $s \notin A$. If A is finite, we use $|A|$ to denote the cardinality of A. As usual, for each $s \in S$ we define $sA = \{st \mid t \in A\}$ and $s^{-1}A = \{t \in S \mid st \in A\}$.

We define a relation R on any semigroup S by sRt for $s, t \in S$ if there exists $x \in S$ with $sx = tx$. If the intersection of finitely many right ideals of S is always nonempty (as when S is left amenable), then R is an equivalence relation, and the set S' of equivalence classes is a right cancellative semigroup with the induced multiplication. More details are found in Granirer [7, p. 371]. When S' exists, we will refer to it as the right cancellative quotient semigroup of S.

Sorenson's conjecture that every right cancellative left amenable semigroup is left cancellative arose as a question of John Sorenson, who proved the weaker result that every right cancellative left