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TAMELY RAMIFIED SUPERCUSPIDAL REPRESENTATIONS OF Gl_n

ROGER E. HOWE

Let F be a non-Archimedean local field of residual characteristic p; then conjecturally the supercuspidal representations of $Gl_n(F)$ are parameterized by admissible characters of extensions of F of degree n provided that n is prime to p. In this paper we establish the existence of the necessary representations if the conjecture is to be true. They will be realized as induced representations from certain subgroups, compact modulo the center. The more difficult question of whether all supercuspidal representations arise by this construction will not be treated. We will also leave aside the problem of computing the characters of these representations.

Let F be a locally compact non-Archimedean field of residual characteristic p. To simplify certain parts of the discussion, we take p to be odd. Let R be a maximal order, π a prime element. Let $F^\times$, $R^\times$ be the multiplicative groups of F and R, and $U = 1 + \pi R \subseteqq R^\times$. Let F' be an extension field of finite degree. We define R', π', $F'^\times$, $R'^\times$, U' in obvious analogy with F. Let $N(F'/F)\colon F'^\times \to F^\times$ be the norm map.

If ψ is a character of $F'^\times$, and $A \subseteqq F'^\times$ is a subgroup, we will say ψ is nondegenerate on A if there is no proper subextension F'', $F \subseteqq F'' \subseteqq F'$, such that $\ker N(F'/F'') \cap A \subseteqq \ker \psi \cap A$.

Now suppose F' is tamely ramified over F. We will say a character ψ of $F'^\times$ is admissible if

(a) ψ is nondegenerate on $F'^\times$, and

(b) if on U', $\psi = \psi'' \circ N(F'/F'')$, where ψ'' is nondegenerate on $U'' \subseteqq F''^\times$, then F' is unramified over F''.

In particular, ψ is admissible if it is nondegenerate on U'.

Given extensions F'_1, F'_2 of F, and characters ψ_i of $F'^\times_i$, we say ψ_1 and ψ_2 are equivalent if there is an F-linear field isomorphism of F_1 onto F_2 which sends ψ_2 to ψ_1.

There are reasons for believing the following conjecture is true.

Conjecture: Suppose n is prime to p. Then the supercuspidal representations of $Gl_n(F)$ are parametrized by admissible characters of extensions of F of degree n. That is, given F' of degree n over F, and ψ an admissible character of $F'^\times$, then one may attach to ψ a supercuspidal representation $V(\psi)$ of $Gl_n(F)$. Two characters correspond to the same representation if and only if they are equivalent. Finally, all supercuspidal representations of $Gl_n(F)$ arise in