

## TOPICS IN HARMONIC ANALYSIS ON SOLVABLE ALGEBRAIC GROUPS

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This paper consists of two parts, and in the first of these we develop the representation theory of solvable algebraic groups over a local field of characteristic zero in analogy with the work of Auslander and Kostant for solvable Lie groups. We show how all the representations arise and show that the Kirillov method of orbits applies to this situation. We find that the theory carries over completely and we discuss traces, CCR representations and we give a version of the Kostant independence of polarization theorem.

In the second part we take up the problems of decomposing the space of square integrable functions on a solvable Lie group modulo a discrete cocompact subgroup. We show how to reduce this problem to the special case when the nilradical of the solvable group is Heisenberg. These two sections represent the initial part of a comprehensive program in this direction to be completed later.

**Introduction.** The theory of harmonic analysis on nilpotent Lie groups was established essentially in one blow by A. A. Kirillov [21] in the early 1960's, after some initial researches by Dixmier. Subsequently, other authors ([16], [17], [24], [29], [37]) explored further topics, such as character theory, extension of these results to unipotent  $p$ -adic groups and more general nilpotent groups, and the spectral decomposition of arithmetic and adelic quotient spaces of unipotent groups. The methods initiated by Kirillov have been extraordinarily supple, and while harmonic analysis on nilpotent groups is hardly complete, I feel it may rightly be considered a mature subject.

Meanwhile, an effort was made to extend Kirillov's analysis to solvable Lie groups. Although the same philosophy eventually succeeded, considerably more sophisticated concepts were required. Several major attacks ([5], [6], [10]) yielded partial progress, but the essence of the problem was not exposed until 1968 with the announcement of Auslander-Kostant [3], and their results, with proofs, have only been published very recently [4]. The Auslander-Kostant results provide a complete description of the representations of a connected, simply connected type I solvable Lie group, and characterize which connected, simply connected solvable Lie groups are type I. More recently, L. Pukanszky [30] has very successfully attacked the problem of doing harmonic analysis on a general, nontype I, solvable Lie