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KIRILLOV THEORY FOR COMPACT p-ADIC GROUPS

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The purpose here is to describe a method by which one may obtain a reasonably explicit and "global" picture of the unitary representation theory of compact p-adic groups, and to indicate some applications. (By p-adic, we refer to Q_p, or local fields of characteristic zero.) The basic inspiration for such a description goes back to Kirillov's work on nilpotent lie groups. The main ingredients are the exponential map and the co-adjoint action. The Campbell-Hausdorff formula is used heavily as a tool.

The basic results indicated here are to some extent implicit in [1], but it seems desirable to have them put in as sharp a form as possible with a view to their application to representations of non-compact groups. Also with this in mind, we have included some facts to allow calculation of intertwining numbers of induced representations. Moreover, since the theory as presented here depends heavily on the exponential map and the Campbell-Hausdorff formula, it is inadequate to deal with all compact p-adic groups. Only "sufficiently compact" groups are covered. Roughly speaking, this means that given a p-adic group, we can analyze completely some open subgroup. Therefore, we have included some propositions showing how the theory of representations of group extensions applies to this situation. These results are all dealt with in section one.

In the succeeding sections we describe some applications. Using Sl_n, we illustrate how compact Cartan subgroups of semisimple groups give rise to series of supercuspidal representations. Then we show how the Kirillov picture can be used to help analyze representations of certain arithmetic groups, like $Sl_n(\boldsymbol{Z})$, $n \geqq 3$. Finally we compute completely the representations of p-adic division algebras of prime degree to show in a simple case how the Kirillov picture can be used as a guide to a full analysis of the representations of a p-adic group. In this section, we place no restriction on the characteristic of the ground field, and the results strongly suggest that although the theory presented here does not apply to groups over fields of positive characteristic, nevertheless the representation theory there is very much like the characteristic zero theory. This, of course, reinforces the experience with Gl_2, Sl_2, etc.

I. Let F be a p-adic field, that is, a finite extension of Q_p,