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SCHUR INDICES OVER THE 2-ADIC FIELD

TOSHIHIKO YAMADA

In this paper it is proved that if G is a finite group with abelian Sylow 2-subgroups, then the Schur index of any character of G over the 2-adic numbers Q_2 is equal to 1. Examples are given so as to show that this statement is false for each odd prime p.

The problem of determining the Schur index of a character of a finite group was reduced by R. Brauer and E. Witt to the case of handling hyper-elementary groups at q, q being a prime. Each of these groups has a cyclic normal subgroup with a factor group which is a q-group. Let p be a prime and Q_p the p-adic numbers. Let G be a hyper-elementary group at q and χ an irreducible character of G. It follows from a result of Witt [1] that if $p = q \neq 2$ then the Schur index $m_{Q_p}(\chi)$ of χ over Q_p is equal to 1. This statement is false for the case $p = q = 2$, because the quaternion group of order 2^3 has an irreducible character χ with $m_{Q_2}(\chi) = 2$.

The purpose of this paper is to show that the above statement also holds for the case $p = q = 2$, provided the Sylow 2-subgroups of a hyper-elementary group at 2 are abelian. In fact, we will prove more generally the following theorem.

THEOREM. *Let G be a finite group with abelian Sylow 2-subgroups. Let χ be any irreducible character of G. Then $m_{Q_2}(\chi) = 1$, that is the Schur index of χ over the 2-adic numbers Q_2 is equal to 1.*

Proof. It is well-known that $m_{Q_2}(\chi) = 1$ or 2 (cf. [1]), so $m_{Q_2}(\chi)$ equals its 2-part. Let n be the exponent of G and let L be the subfield of $Q_2(\zeta_n)$, ζ_n a primitive nth root of unity, such that $L \supset Q_2(\chi)$, $2 \nmid [L: Q_2(\chi)]$ and $[Q_2(\zeta_n): L]$ is a power of 2. By the Brauer-Witt theorem [3, p. 31] there is an L-elementary subgroup H of G with respect to 2 and an irreducible character θ of H with the following properties: (1) there is a normal subgroup N of H and a linear character ψ of N such that $\theta = \psi^H$; (2) $H/N \cong \mathrm{Gal}(L(\psi)/L)$, in particular, H/N is a 2-group; (3) $L(\theta) = L$; (4) $m_L(\theta) = m_L(\chi) = m_{Q_2(\chi)}(\chi) = m_{Q_2}(\chi)$; (5) for every $h \in H$ there is a $\tau(h) \in \mathrm{Gal}(L(\psi)/L)$ such that $\psi(hnh^{-1}) = \tau(h)(\psi(n))$ for all $n \in N$; (6) $m_L(\theta)$ is the index of the crossed product $(\beta, L(\psi)/L)$ where, if D is a complete set of coset representatives of N in H $(1 \in D)$ with $hh' = n(h, h')h''$ for $h, h', h'' \in D$, $n(h, h') \in N$, then $\beta(\tau(h), \tau(h')) = \psi(n(h, h'))$. Since ψ is