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TORSION FREE ABELIAN GROUPS QUASI-PROJECTIVE OVER THEIR ENDOMORPHISM RINGS II

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Let R be a commutative ring with 1, and X an R-module. Then $M = X \oplus R$ is quasi-projective as an E-module, where E is either $\mathrm{Hom}_Z(M, M)$ or $\mathrm{Hom}_R(M, M)$. In this note it is shown that any torsion free abelian group G of finite rank, quasi-projective over its endomorphism ring, is quasi-isomorphic to $X \oplus R$, where R is a direct sum of Dedekind domains and X is an R-module.

Introduction. If R is a ring with identity, an R-module M is said to be quasi-projective if for any submodule N of M, and R-map $f: M \to M/N$, there is an R-map $\bar{f}: M \to M$ such that $\bar{f}$ followed by the factor map $M \to M/N$ is equal to f. Results on quasi-projective modules appear in [3], [6], and [7]. In this note, we will be concerned with the case where $M = G$ is a torsion free abelian group of finite rank and $R = \mathrm{Hom}_Z(G, G) = E(G)$, and call G "*Eqp*" if G is quasi-projective as an $E(G)$-module. The strongly indecomposable *Eqp* groups have been characterized in [6], so we will focus on those groups G (always torsion free abelian of finite rank) such that $nG \subseteq G_1 \oplus G_2 \subseteq G$ for some integer $n \neq 0$ and subgroups $G_1 \neq 0$, $G_2 \neq 0$ of G. In fact, any group G can be quasi-decomposed into a direct sum of strongly indecomposable summands, $nG \subset G_1 \oplus G_2 \oplus \cdots \oplus G_k \subset G$. It is well-known that such a decomposition is unique up to order and the quasi-isomorphism class of the summands. It is therefore desirable to work with a slightly more general notion of quasi-projectivity which is invariant under quasi-isomorphism:

DEFINITION. An R-module M is almost quasi-projective (*aqp*) if there exists an integer $n \neq 0$ such that given any submodule N of M, and R-map $f: M \to M/N$, there is an R-map $\bar{f}: M \to M$ such that $\bar{f}$ followed by the factor map $M \to M/N$ is equal to nf.

In case M is a group G and $R = E(G)$, G is called *almost E-quasi-projective* (*aEqp*).

PROPOSITION 1. *Let G and H be quasi-isomorphic groups* (notation: $G \sim H$). *If G is aEqp, then H is aEqp.*

Proof. Assume that $mG \subseteq H \subseteq G$ for some integer $m \neq 0$. Then if $\alpha \in E(G)$, $m\alpha|_H \in E(H)$; and if $\beta \in E(H)$, $\beta m \in E(G)$, so we say