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FIXED POINT SETS OF PEANO CONTINUA

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For each positive integer $n = 1, 2, \cdots$, it is shown that there is an $(n+1)$-dimensional acyclic LC^{n-1} continuum X_n containing an n-dimensional sphere which is not the fixed point set of any self-map of X_n.

1. Introduction. A subset A of a topological space X is called a *fixed point set* of X if there is a (continuous) map $f: X \rightarrow X$ such that $f(x) = x$ iff $x \in A$. If X is Hausdorff, then A is closed, and, clearly, every retract of X is a fixed point set of X. It is possible that a space X may have the property that each of its nonempty closed subsets is a fixed point set of X. The problem of determining which spaces have this property, called the *complete invariance property* by L. E. Ward, Jr. in [5], has been investigated by H. Robbins, Helga Schirmer, and L. E. Ward, Jr. Some spaces known to have the complete invariance property include n-cells [1], dendrites [2], convex subsets of Banach spaces [5], compact manifolds without boundary [3], and all compact triangulable manifolds with or without boundary [4].

The general question as to what properties a space must satisfy to insure that it has the complete invariance property has not been resolved. In fact, in [5, p. 553] L. E. Ward, Jr. asks the following question.

Does every Peano continuum have the complete invariance property?

The purpose of this note is to show that even acyclic Peano continua which possess higher order local connectedness need not have the complete invariance property. Indeed, for each positive integer $n = 1, 2, \cdots$, we give an example of an $(n+1)$-dimensional acyclic LC^{n-1} continuum X_n which fails to have the complete invariance property. Moreover, X_n contains an n-dimensional sphere which is not a fixed point set of X_n.

2. Notation and the construction of X_n. We shall let E^n denote Euclidean n-dimensional space, and we shall consider E^m to be canonically imbedded in E^n if $m < n$. The closed unit ball in E^{n+1} shall be denoted by B^{n+1} and the boundary $\operatorname{Bd} B^{n+1}$ of B^{n+1} shall be denoted by S^n.

Consider the rectangle in $E^2 \subset E^{n+2}$ with vertices $(1, -1, 0, \cdots 0)$, $(0, -1, 0, \cdots, 0)$, $(0, 1, 0, \cdots, 0)$, $(1, 1, 0, \cdots, 0)$. Let D denote the closed disk consisting of this rectangle together with its interior in