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# A NASH-MOSER-TYPE IMPLICIT FUNCTION THEOREM AND NON-LINEAR BOUNDARY VALUE PROBLEMS

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**The main objective of this paper is to formulate an implicit function theorem for Frechet spaces which is suitable for nonlinear systems of partial differential equations with prescribed boundary conditions. The applications are discussed in connection with deformation theory where such problems arise naturally and are of fundamental importance. Furthermore, their linearizations are certain second-order perturbations of second-order elliptic noncoercive boundary value problems. The last part of the paper deals with developing a general theory which covers these cases.**

In [12] J. Moser gives a rather general method for the construction of solutions of nonlinear differential equations whose linearizations lose derivatives. His result is similar to that first formulated by J. Nash in [12] in connection with the isometric embedding of Riemannian manifolds. The recent progress in the theory of pseudo-complex structures has further emphasized the importance of the Nash-Moser technique. Various generalizations of this approach have been successfully used by R. Hamilton for the study of certain nonlinear complexes of partial differential operators (cf. [2]). Hamilton's version of a Nash-Moser-type inverse function theorem has enabled M. Kuranishi to construct a finite-dimensional universal family of deformations of pseudo-complex structures on a strongly pseudo-convex pseudo-complex compact manifold (cf. [10]).

One distinctive feature of the nonlinear problems which appear in the areas mentioned so far is that they are free of boundary conditions, although, as it has been demonstrated by R. Hamilton, the construction of inverses of the linearizations is very often achieved by considering linear boundary value problems. For example, the deformation theory developed by M. Kuranishi in [10] takes place on a compact $C^\infty$ manifold $M_0$ which is the boundary of a complex manifold $M$. Hence the relevant nonlinear systems of partial differential equations naturally have no boundary conditions imposed on them. On the other hand, it is shown in [2] that if $H^1(M, T') = 0$, where $T'$ is the holomorphic tangent bundle, and the Levi form on $M_0$ never has exactly one negative eigenvalue, then for any complex structure $M_\omega$ on $M$ sufficiently close to the given structure one can find a $C^\infty$ diffeomorphism $f$ of $M$ into the ambient manifold $M'$ so that $f: M_\omega \to M'$ is complex analytic. Here $\omega$ is a